Tests of Hypotheses

1. **Test a Population Mean**

   Null Hypothesis: $H_0\colon \mu = \mu_0$

   Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon \mu > \mu_0$	$z > z_\alpha$ (upper-tailed test)
$H_a\colon \mu < \mu_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a\colon \mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have a normal population, the sample size should be large i.e., $n \geq 30$. If the population standard deviation σ is not known, we use the sample standard deviation s, in this case we need $n \geq 30$ even if the population is normal.

2. **Test a Population Proportion**

   Null Hypothesis: $H_0\colon p = p_0$

   Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon p > p_0$	$z > z_\alpha$ (upper-tailed test)
$H_a\colon p < p_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a\colon p \neq p_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample size should be large i.e., $np_0 > 5$ and $nq_0 > 5$.

3. **Test a Difference Between two Population Means**

   Null Hypothesis: $H_0\colon \mu_1 - \mu_2 = D_0$

   Test Statistic: $z = \dfrac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon \mu_1 - \mu_2 > D_0$	$z > z_\alpha$ (upper-tailed test)
$H_a\colon \mu_1 - \mu_2 < D_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a\colon \mu_1 - \mu_2 \neq D_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have normal populations, the sample sizes should be large i.e., $n_1 \geq 30$ and $n_2 \geq 30$, and the two samples should be independently randomly selected. If σ_1 and σ_2 are not known, we use s_1 and s_2, in this case we need $n_1 \geq 30$ and $n_2 \geq 30$.

4. **Test a Difference Between two Population Proportions**

   Null Hypothesis: $H_0\colon p_1 - p_2 = 0$

   Test Statistic: $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(1/n_1 + 1/n_2)}}$, where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$.

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon p_1 - p_2 > 0$	$z > z_\alpha$ (upper-tailed test)
$H_a\colon p_1 - p_2 < 0$	$z < -z_\alpha$ (lower-tailed test)
$H_a\colon p_1 - p_2 \neq 0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample sizes should be large i.e., $n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2$, and $n_2\hat{q}_2$ should all be greater than 5, and the two samples should be independently randomly selected.

5. **p-value**

   The p-value is the smallest value of α for which H_0 could be rejected. It is the probability that the null hypothesis could produce an observed sample at least as extreme as the one that was observed. The smaller the p-value, the stronger the evidence against H_0.

   For an upper-tailed test, the p-value is $P(Z > z_{\text{obs}})$.

   For a lower-tailed test, the p-value is $P(Z < z_{\text{obs}})$.

   For a two-tailed test, the p-value is $P(Z > |z_{\text{obs}}|) + P(Z < -|z_{\text{obs}}|) = 2\,P(Z > |z_{\text{obs}}|)$.

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