Poker Hands

by Gilles Cazelais

It is a good exercise in counting to count the number of poker hands of each type.

The total number of five-card poker hands is $\binom{52}{5} = 2\,598\,960$.

A **royal flush** consists of the 10, Jack, Queen, King, and Ace of the same suit. There are 4 royal flushes, one for each suit.

A **straight flush** consists of five cards of the same suit in a consecutive sequence that is not a royal flush. It is completely determined by the smallest card in the straight. For a *nonroyal* straight flush of a given suit, there are 9 choices for the smallest card. Hence, there are $9 \cdot 4 = 36$ straight flushes.

A **4-of-a-kind** is a hand with four cards of the same kind. The number of such hands is $\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1} = 624$.

A **full-house** consists of three cards of one kind and two of another. The number of full-houses is $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3\,744$.

A **flush** consists of five cards from the same suit, excluding straight or royal flushes. The number of flushes is $\binom{13}{5}\binom{4}{1} - 40 = 5\,108$.

A **straight** consists of five cards in a consecutive sequence, not all of the same suit. The smallest card of a straight can be one of the 10 kinds: Ace, $2, 3, \ldots, 10$. For each of the 5 cards, we have 4 choices of suits. The number of straights (nonflush) is then $10 \cdot 4^5 - 40 = 10\,200$.

A **3-of-a-kind** is a hand with three cards of the same kind. The number of such hands is $\binom{13}{1}\binom{4}{3}\binom{12}{2} \cdot 4^2 = 54\,912$.

The number of hands with **two-pairs** is $\binom{13}{2}\binom{4}{2}\binom{4}{2} \cdot 44 = 123\,552$. Note that once you have selected the two pairs, there are 44 choices for the fifth card.

The number of hands with one **pair** is $\binom{13}{1}\binom{4}{2}\binom{12}{3} \cdot 4^3 = 1\,098\,240$.

A **high-card** hand has five cards of distinct kinds, not in a consecutive sequence, and not all of the same suit. There are $\binom{13}{5} - 10$ ways of choosing five distinct kinds not in a sequence. Since any choice of suits is acceptable except the 4 where all five cards are of the same suit, we have $4^5 - 4$ choices. The number of high-card hands is then

$$\left(\binom{13}{5} - 10\right)(4^5 - 4) = 1\,302\,540.$$

Hand name	Number of hands	Probability
royal flush	4	0.0000015
straight flush	36	0.0000138
4-of-a-kind	624	0.00024
full-house	3 744	0.00144
flush	5 108	0.00197
straight	10 200	0.00392
3-of-a-kind	54 912	0.0211
two-pairs	123 552	0.0475
pair	1 098 240	0.4226
high-card	1 302 540	0.5011
Total	2 598 960	1

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