

Solution of a Linear System

Consider the following linear system of equations.

$$3x + y = 5$$

$$x + 4y = 9$$

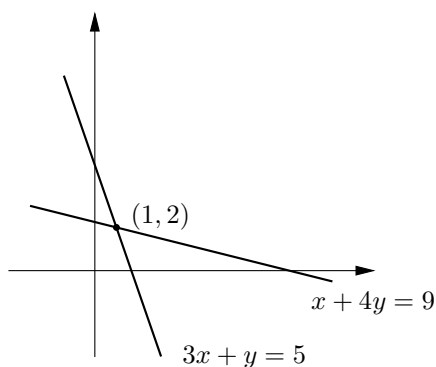
We can solve it to obtain the solution

$$x = 1 \quad \text{and} \quad y = 2.$$

Let's now interpret geometrically this solution in two different ways. The second one is very important in Linear Algebra.

Row interpretation.

Each row in the linear system corresponds to the equation of a straight line. The solution corresponds to the point in the xy -plane where the two lines intersect.



Column interpretation.

The system of equations can be expressed in column vector form as follows.

$$x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

Let's define

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}.$$

The solution $x = 1$ and $y = 2$ tells us that we can express vector \mathbf{b} as a linear combination of vectors \mathbf{v}_1 and \mathbf{v}_2 as follows.

$$\mathbf{v}_1 + 2\mathbf{v}_2 = \mathbf{b}$$

