MATH 155: Sample Final Questions - Sequences & Series

Math 173 – Assignment #6 **Name:** Solution Set

Total: 40

1. Find all terms of the finite sequence $a_n = \frac{(-1)^n}{n!}, 1 \le n \le 4$.

$a_1 = \frac{(-1)^1}{1!} = 1$ $a_3 = \frac{(-1)^3}{3!} = \frac{-1}{6}$ (2)

$a_2 = \frac{(-1)^2}{2!} = \frac{1}{2}$ $a_4 = \frac{(-1)^4}{4!} = +\frac{1}{24}$

$-1, \frac{1}{2}, -\frac{1}{6}, \frac{1}{24}$

2. State whether the following are arithmetic sequences, geometric sequences, or neither. Also, give a formula for the nth term of the sequence. Start with index of one.

a) 15, 9, 3, -3, …

$a_1 = 15$

$d = -6$

$a_n = a_1 + (n-m)d$

$a_n = a_1 + (n-1)d$

$= 15 + (n-1)(-6)$

$= 15 - 6n + 6$

$= 21 - 6n$

arithmetic

either $a_n = 21 - 6n$ (2)

or $\begin{cases} a_1 = 15 \\ a_n = a_{n-1} - 6 \end{cases}$

b) $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$ $\frac{n-1}{n}$

1 2 3 4 5 n

neither

$a_n = \frac{n-1}{n}$ (2)

c) 48, 12, 3, ¾, …

$a_1 = 48$

$r = \frac{1}{4}$

$a_n = a_m r^{n-m}$

$a_n = a_1 r^{n-1}$

$= 48\left(\frac{1}{4}\right)^{n-1}$

$\left[= 48\left(\frac{1}{4}\right)^n\left(\frac{1}{4}\right)^{-1} \right.$

$\left. = \frac{192}{4^n} \right]$ if you insist

geometric

either $a_n = 48\left(\frac{1}{4}\right)^{n-1}$ (2)

or $\begin{cases} a_1 = 48 \\ a_n = \frac{1}{4} a_{n-1} \end{cases}$

3. Write the first three terms of the infinite sequence given by the recursion formula

$$\begin{cases} a_1 = 2 \\ a_n = (a_{n-1})^2 + 1 \end{cases} \quad \text{for } n \geq 2$$

$2, 5, 26, \ldots$ (3)

$a_2 = a_1^2 + 1 = 2^2 + 1 = 5$

$a_3 = a_2^2 + 1 = 5^2 + 1 = 26$

4. Find the sum of the following series.

a) $\sum_{j=0}^{4}(3j) = \underset{j=0}{0} + \underset{j=1}{3} + \underset{j=2}{6} + \underset{j=3}{9} + \underset{j=4}{12}$ 30 (2)

$= 30$

b) $2 + 4 + 6 + \ldots + 88$ 1980 (3)

arithmetic with

$a = 2$ and $d = 2$

so $S_n = \frac{n}{2}(a_1 + a_n)$

but what's n?

$a_n = a_m + (n-m)d$

$a_n = a_1 + (n-1)d$

$88 = 2 + (n-1)2$

$86 = 2(n-1)$

$43 = n-1$

$n = 44$

$S_n = \frac{44}{2}(2+88)$

$= 1980$

c) $\sum_{i=0}^{\infty} 300(0.99)^i = \underset{i=0}{300} + \underset{i=1}{300(0.99)} + \underset{i=2}{300(0.99)^2} \ldots$ 30000 (2)

geometric with

$a_m = 300$

$r = 0.99$ and $|r| < 1$ ✓

so $S_\infty = \frac{a_m}{1-r} = \frac{300}{1-0.99}$

$= 30000$

d) $\frac{1}{25} - \frac{1}{20} + \frac{1}{16} - \frac{5}{64} + \ldots$ undefined (1)

geometric with

$a_1 = 1/25$

$r = -5/4$ ← but $\cancel{|r| < 1}$ not satisfied