MATH 155: Sample Final Questions – Sequences and Series

6. Label the following sequences as "arithmetic", "geometric" or "neither". (3 points)

   a) 58, 48, 38, … add -10 arithmetic

   b) 1, 1, 2, 3, 5, 8, … Fibonacci neither

   c) $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \ldots$ mult by $\frac{1}{3}$ geometric

7. Consider the sequence given by the following. (4 points)

$$a_n = 30 - 3n, \qquad 1 \le n \le 3$$

   a) Is this formula recursive or general? (Circle one.) recursive / general

   b) Calculate all terms of this sequence. 27, 24, 21

$a_1 = 30 - 3(1) = 27$

$a_2 = 30 - 3(2) = 24$

$a_3 = 30 - 3(3) = 21$

(−1½) if give sum of series (it's a sequence)

8. Evaluate the following sum, if it exists. Show your work! (2 points)

$\sum_{i=2}^{\infty} 8(-3)^i = 8(-3)^2 + 8(-3)^3 + 8(-3)^4 + \ldots$

geometric with $r = -3$

$|r| < 1$? No!

undefined

9. Calculate the first three terms of the following sequence. (3 points)

$$\begin{cases} a_1 = 3 \\ a_n = (a_{n-1})^2 \end{cases}$$ for $n \ge 2$

$a_2 = a_1^2 = 3^2 = 9$

$a_3 = a_2^2 = 9^2 = 81$

3, 9, 81

10. Write a recursive formula for the sequence defined below and draw a box around your answer. (2 points)

$a_n = 7\cdot 3^n$ for $n \geq 1$

$a_1 = 7\cdot 3^1 = 21$

$a_2 = 7\cdot 3^2 = 63$

$a_3 = 7\cdot 3^3 = 189$

$$\boxed{\begin{array}{l} a_1 = 21 \\ a_n = 3a_{n-1} \text{ for } n \geq 2 \end{array}}$$

geometric with r = 3

11. Calculate the sum of the odd numbers between 1000 and 5000. Be sure to show your work. (4 points)

$1001 + 1003 + 1005 + \ldots 4999$

arithmetic series with $d = 2$ (1)

number of terms:

$a_n = a_m + (n-m)d$

Let's start with m = 1 →

$a_n = a_1 + (n-1)d$

$4999 = 1001 + (n-1)\cdot 2$

$3998 = (n-1)2$

$1999 = n-1$

$n = 2000$ (2)

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{2000} = \frac{2000}{2}(1001 + 4999)$

$$\boxed{S_{2000} = 6\,000\,000}$$ (1)

if n correct but no work, (-1)

if got n by last - first + 1 (-2)

if assumed $S_{odd} = S_{even}$ (-2)

if used even numbers (-1) and sum is $S_{2001} = 6\,003\,000$