Section 1.1: Decimal and Octal

Tuesday, September 3, 2019 10:24 AM

decimal: base 10 (why? ten fingers)

historical note: (will not be tested)

Sumerians / Babylonians used base 60

Mayans used base 20

Oksapmin used base 27

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decimal:

0
1
2
3
4
5
6
7
8
9
10 ← one ten, no ones
11 ← one ten, one one
12 ← one ten, two ones

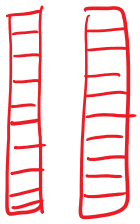
note: in base 10, or any base, we use two symbols to represent the base

- we don't have a single symbol for "ten"

the number 23 means:

2 tens
2 of the base

3 ones
3 leftover



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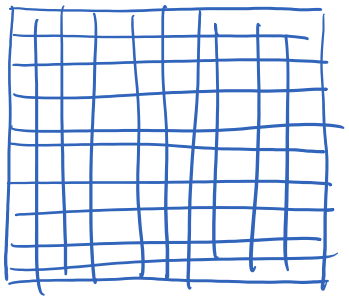
what about 123?

1 hundred



$123 = 1 \times 100 + 2 \times 10 + 3 \times 1$

$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

for bases other than ten, it's the same idea!

let's look at base 4:

0

1

2

3

10_4

11_4

12_4

13_4

20_4

21_4

22_4

23_4

30_4

31_4

32_4

33_4 ← 3 fours, 3 ones

100_4 ← 1 sixteen, no fours, no ones

↑ group of 4 fours

$4^2 = 16$

notation: 123_4

↖ subscript is the base

if no subscript, default is 10

$$123_4 = 1 \times 4^2 + 2 \times 4^1 + 3 \times 4^0$$

$$= 16 + 8 + 3$$

$$= 27 \text{ or } 27_{10}$$

example: convert the following numbers to decimal

a) 213_4 : $2 \times 4^2 + 1 \times 4^1 + 3 \times 4^0 = 39$

b) 3012_4 : $3 \times 4^3 + 0 \times 4^2 + 1 \times 4^1 + 2 \times 4^0 = 198$

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what bases are commonly used in computing?

binary - base 2
octal - base 8
hexadecimal - base 16

octal: base 8

decimal	octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10_8
9	11_8
10	12_8
11	13_8
12	14_8
13	15_8
14	16_8

13	15_8
14	16_8
15	17_8
16	20_8

example: convert to decimal:

a) $72_8 = 7 \times 8^1 + 2 \times 8^0 = 58$

b) 5604_8 : $5 \times 8^3 + 6 \times 8^2 + 0 \times 8^1 + 4 \times 8^0 = 2948$

example: convert to decimal:

a) $212_3 = 2 \times 3^2 + 1 \times 3^1 + 2 \times 3^0 = 23$

example: what's wrong with writing 215_3 ?

working in base 3, only allowed digits are 0, 1, 2

5 is not allowed

note: in base 10, there are 10 digits (0, 1, 2, 3, ... 9)

4 4 (0, 1, 2, 3)

8 8 (0, 1, 2, 3, ... 7)