Section 2.3: Logical Equivalence

Thursday, September 12, 2019 10:22 AM

Consider the propositions p and q:

There are four possible combinations of values for p and q because each of them can be either true or false

truth table: (extremely long-winded version)

p	q	p ^ q
false	false	false
false	true	false
true	false	false
true	true	true

(or could write F and T)

really terse version

↑ will use this!

let 0 = false
1 = true

p	q	p ^ q
0	0	0
0	1	0
1	0	0
1	1	1

truth tables:

p	q	$p \wedge q$	$p \vee q$	$\bar{p}$	$\overline{p \vee q}$
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	0	0

example: write the truth table for $p \oplus q$

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

example: write the truth table for $\overline{p \vee \bar{q}} \wedge \bar{r}$

p	q	r	$\bar{q}$	$\bar{r}$	$p \vee \bar{q}$	$\overline{p \vee \bar{q}}$	$\overline{p \vee \bar{q}} \wedge \bar{r}$
0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

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example: write the truth table for $p \wedge \bar{p}$

p	$\bar{p}$	$p \wedge \bar{p}$
0	1	0
1	0	0

and from this table, we can see that

$$p \wedge \bar{p} \iff 0$$

↑ "is logically equivalent to"

⇒ so we can use truth tables to simplify logical expressions

example: use a truth table to simplify $p \wedge 1$

p	1	$p \wedge 1$
0	1	0
1	1	1

↑ same ↑

conclusion:

$$p \wedge 1 \iff p$$

simplify $(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{q})$

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \wedge \bar{q}$	$p \wedge \bar{q}$	$(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{q})$

p	q	$\bar{p}$	$\bar{q}$	$p \wedge q$	$p \wedge q$	$(\bar{p} \wedge q) \vee (p \wedge \bar{q})$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

Same

$\bar{q}$

or $(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{q}) \iff \bar{q}$

Is $\overline{p \oplus q}$ logically equivalent to $\bar{p} \oplus \bar{q}$?

p	q	$\bar{p}$	$\bar{q}$	$p \oplus q$	$\overline{p \oplus q}$	$\bar{p} \oplus \bar{q}$
0	0	1	1	0	1	0
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

NO

or $\overline{p \oplus q} \not\iff \bar{p} \oplus \bar{q}$