Section 2.5: Laws of Logic (LoL)

Monday, September 16, 2019 11:16 AM

there are connections between logic & Boolean algebra:

logic	$p \wedge q$	$p \vee q$	$\bar{p}$	F (false)	T (true)
Boolean algebra	AB	$A+B$	$\bar{A}$	0	1

we can show that

$p \wedge 1 \Leftrightarrow p$ by using a truth table

p	1	$p \wedge 1$
0	1	0
1	1	1

$A \cdot 1 = A$

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these statements are true for all possible variables, so we call them laws – in particular, this one is called an identity law

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identity laws: there are four of them

$$p \wedge 1 \Leftrightarrow p$$
$$p \vee 1 \Leftrightarrow 1$$
$$p \wedge 0 \Leftrightarrow 0$$
$$p \vee 0 \Leftrightarrow p$$

but note these statements are true for <u>all</u> possible variables

since $p \vee 0 \Leftrightarrow p$, then

$$\bar{q} \vee 0 \Leftrightarrow \bar{q}$$

$$r \vee 0 \Leftrightarrow r$$

$$☺ \vee 0 \Leftrightarrow ☺$$

$$\overline{p \wedge q} \vee 0 \Leftrightarrow \overline{p \wedge q}$$

why do we care?

example: simplify the following using the laws of logic

- use only one law per line
- state the name of the law you are using

$$(p \wedge 0) \vee (p \wedge 1)$$

$$0 \vee p \qquad \text{identity}$$

$$p \qquad \text{identity}$$

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idempotent

$$p \wedge p \Leftrightarrow p$$

$$p \vee p \Leftrightarrow p$$

which also means that $\bar{r} \wedge \bar{r} \Leftrightarrow \bar{r}$

$\overline{p \wedge r} \wedge \overline{p \wedge r} \Leftrightarrow \overline{p \wedge r}$

complement:

$$\bar{\bar{p}} \Leftrightarrow p$$

$$p \wedge \bar{p} \Leftrightarrow 0$$

$$p \vee \bar{p} \Leftrightarrow 1$$

examples: simplify the following using the complement law:

a) $q \wedge \bar{q} \Leftrightarrow 0$

b) $ABC + \overline{ABC} = 1$

c) ☺ $\cdot$ $\overline{☺}$ $= 0$

note: for MATH 155, you may omit explicitly writing the commutative and/or associative laws using a separate step

example: simplify $0 \vee p$

nitpicker solution

	$0 \vee p$	
	$p \vee 0$	commutative
	p	identity

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	$0 \vee p$	
	p	identity

totally acceptable MATH 155 solution

$0 \vee p$

p identity

simplify using the LOL:

$(\bar{p} \vee 0) \wedge (q \vee \bar{q}) \wedge (1 \vee r)$

$\bar{p} \wedge (q \vee \bar{q}) \wedge 1$ identity

$\bar{p} \wedge 1 \wedge 1$ complement

$(\bar{p} \wedge 1) \wedge 1$ note: can skip this step but it's associative

$\bar{p} \wedge 1$ identity

$\bar{p}$ "

$(p \wedge \bar{p}) \vee (p \vee \bar{p})$

$0 \vee 1$ complement

1 one of $\left\{ \begin{array}{l} \text{identity} \\ \text{complement} \\ \text{definition of "or"} \end{array} \right.$

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simplify:

$\overline{p \vee (q \wedge \bar{r})} \wedge (p \vee (q \wedge \bar{r}))$

(brainteaser)

0 complement

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simplify:

$A(\bar{B}B)$	$+$	$B(A+\bar{A})$		
$A \cdot 0$	$+$	$B \cdot 1$		complement
0	$+$	B		identity
	B			"

example: write a simplified expression for each of the following and state the name of the law you've used

a) $\bar{F} \wedge \bar{F} \Leftrightarrow \bar{F}$ idempotent

b) $ABC + ABC = ABC$ "

c) $ABC + \overline{ABC} = 1$ complement

d) $0 + \bar{B} = \bar{B}$ identity

summary:

identity laws : deal with zeros or ones

idempotent laws : deal with a variable and/or itself

complement laws : deal with negations

commutative
associative

can omit