Section 2.7: The Conditional

Thursday, September 19, 2019 10:44 AM

Conditional:

$$p \rightarrow q$$

"p implies q"

"if p, then q"

example: If I live in Saanich, then I live in BC.

→ who's breaking the law handout

example: The following statement is true:

"If Barney is a dog, then Barney has four legs."

Answer the following questions with "yes", "no", or "maybe".

a) Barney is a dog. Does he have four legs? Y
b) Barney is not a dog. Does he have four legs? M
c) Barney has four legs. Is he a dog? M
d) Barney does not have four legs. Is he a dog? N

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true: "If Snarks are Boojums ($p \rightarrow$), then the Bellman is incorrect (q)."

true: "If Snarks are Boojums, then the Bellman is incorrect."

which of the following cannot occur?

p is true

a) Snarks are Boojums and the Bellman is incorrect.
b) Snarks are not Boojums and the Bellman is incorrect.
c) Snarks are not Boojums and the Bellman is correct } q false
d) Snarks are Boojums and the Bellman is correct.

p true

you can't have the first one true and the second one false

2019/09/25

the conditional: $p \rightarrow q$

p	q	p → q
0	0	1
0	1	1
1	0	0
1	1	1

If I live in Saanich, then I live in BC.

example: If Pat sleeps in, then she will be late for class.

$p \rightarrow q$, the conditional

for this conditional,

$q \to p$ is called the converse

is the conditional logically equivalent to the converse?

converse: "If Pat was late for class, then Pat slept in."

this statement is not logically equivalent to the conditional!

how do we prove it? truth table

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

$p \to q$ is not logically equivalent to $q \to p$

$(p \to q) \not\Leftrightarrow (q \to p)$

example: for the conditional $p \to q$, is it logically equivalent to the contrapositive $\bar{q} \to \bar{p}$?

p	q	$\bar{p}$	$\bar{q}$	$p \to q$	$\bar{q} \to \bar{p}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

Yes

$(p \to q) \Leftrightarrow (\bar{q} \to \bar{p})$

example: write the contrapositive $\bar{q} \to \bar{p}$ for the following conditional $p \to q$

If I live in Saanich or Esquimalt, then I live in BC.

answer: If I don't live in BC, then I don't live in Saanich AND I don't live in Esquimalt.

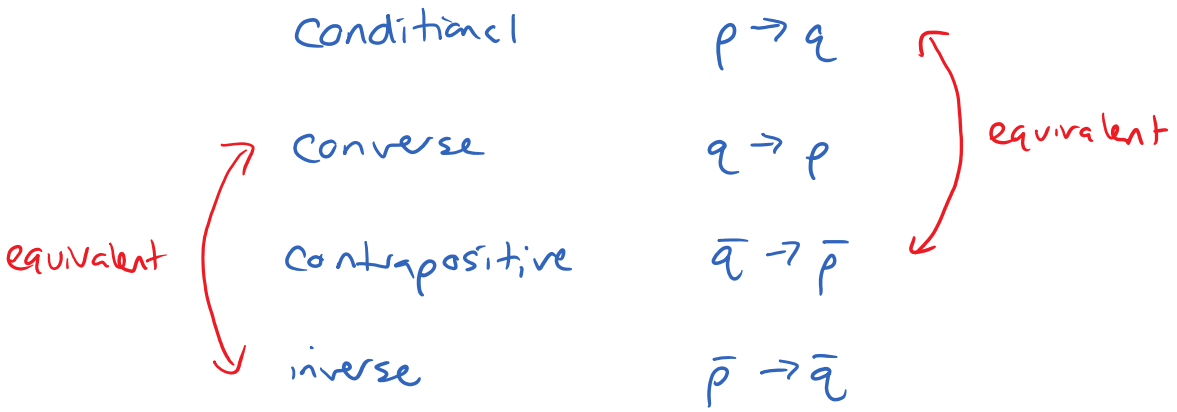
↑ stealth DeMorgan's !

Why? $\overline{\text{Saanich or Esquimalt}} \Leftrightarrow \overline{\text{Saanich}}$ AND $\overline{\text{Esquimalt}}$

another perfectly acceptable answer:

If I don't live in BC, then I live in NEITHER Saanich NOR Esquimalt

what about the inverse, $\bar{p} \to \bar{q}$?



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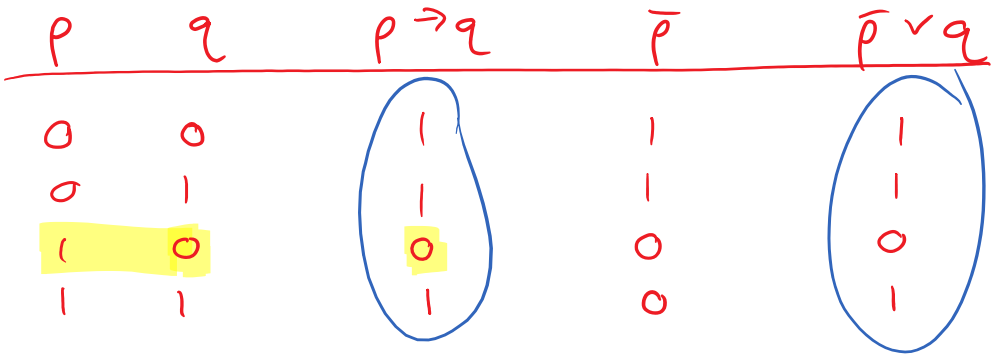
the "or" form of the conditional:



$p \to q \iff \bar{p} \vee q$

example: for the following conditionals $p \to q$, rewrite them in the "or" form, $\bar{p} \vee q$

a) If I live in Saanich, then I live in BC.

b) If Pat sleeps in, then she will be late for class.

answers:

a) I don't live in Saanich or I live in BC (or both).

b) Pat didn't sleep in or she was late for class (or both).

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digression: will not be tested

Why do we care?

pseudocode:

```
if x > 3 then y = 4
print y
```

question: if the output is "4", was $x > 3$?

consider:

```
x = 5
y = 7

if x > 3 then y = 4
print y
```

output: 4

```
x = 2
y = 4

if x > 3 then y = 4
print y
```

output = 4

↑ ↗

same!

but $x > 3$ only in left-hand case