

Section 2.8: The Biconditional

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biconditional:

If you get 90% or higher in Math 155,
then you will get an A+ and vice versa.

or

If and only if you get 90% or higher
in Math 155, then you will get an A+

what does it mean?

"if p , then q and if q , then p "

or "if p , then q and if \bar{p} , then \bar{q} "

or "either p and q are both true or
they are both false"

notation:

$$p \leftrightarrow q$$

"if and only if p , then q " (iff p , then q)

"if p , then q and vice versa"

example:

Consider the following conditional statements.
Do they still make sense when written
as a biconditional?

① If you eat at Joe's, you will have a good meal.

No

② If two lines are perpendicular, then they meet at a right angle.

Yes

③ If today is February 14th, then today is Valentine's Day.

Yes

④ If the car battery is drained, then the car will not start.

No

truth table:

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

example: Is the biconditional $p \leftrightarrow q$ logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$?
Use a truth table to justify your answer.

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
0	0	1	1	1	1

0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

Yes

$$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$$

example: The following statement is true:

If and only ^p Snarks are Boojums, then the ^q Bellman is incorrect.

- a) Snarks are Boojums. Is the Bellman correct? No
p is true so q is true
- b) Snarks are not Boojums. Is the Bellman correct? Yes
p is false q is false
- c) The Bellman is correct. Are Snarks Boojums? No
- d) The Bellman is incorrect. Are Snarks Boojums? Yes

either p and q are both true or they are both false