Section 3.1: Sequences and Series

Thursday, September 26, 2019 11:03 AM

Sequence $\equiv$ an ordered set of numbers (often with a pattern)

is defined as

examples:

	pattern?
a) $2, 5, 8, \ldots$	add 3
b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \frac{1}{256}$	mult by $\frac{1}{2}$
c) $1, 4, 9, 16, 25, \ldots 100$	n^2
d) $5, -5, -15, \ldots$	add -10

a and d are infinite sequences
b and c are finite

notation:

$a_1, a_2, a_3, \ldots a_n, \ldots$

if you start counting from 1

or

$a_0, a_1, a_2, \ldots a_n, \ldots$

what is the previous term a_{n-1}

if you start counting from zero

or

$a_m, a_{m+1}, a_{m+2}, \ldots a_n, \ldots$

$= \quad a_m, \quad a_{m+1}, \quad a_{m+2}, \ldots \quad a_n, \ldots$

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three ways to define a sequence:

① list all of the terms or
list enough terms to set up the pattern
(minimum is three, though for complicated patterns you may need far more)

note: if the sequence is finite, then you need to provide either the final term or the total number of terms

② give a general formula for a_n

③ give a recursive formula for a_n

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general formula: formula that gives a_n by way of n only

example: $\boxed{a_n = 3n-1 \quad \text{for} \quad n \geq 1}$

what are the first three terms of the above sequence?

$a_1 = 3(1) - 1 = 2$

$a_2 = 3(2) - 1 = 5$
$a_3 = 3(3) - 1 = 8$

what's a_{100}?

$$a_{100} = 3(100) - 1 = 299$$

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example: write all terms of the sequence

$$a_n = 2^n + 1 \quad \text{for} \quad 0 \le n \le 4$$

answer:
$a_0 = 2^0 + 1 = 2$
$a_1 = 2^1 + 1 = 3$
$a_2 = 2^2 + 1 = 5$
$a_3 = 2^3 + 1 = 9$
$a_4 = 2^4 + 1 = 17$

note: how many terms in total are there?

if k is the total number of terms,

$$k = \underset{\substack{\text{final}\\\text{index}}}{n} - \underset{\substack{\text{first}\\\text{index}}}{m} + 1$$

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example: what is the n^{th} term for the sequence

1 2 3 4 5 6 ← write the index of term

1 2 3 4 5 6

$1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \ldots$ ← write the index of term directly above

$$\boxed{a_n = \sqrt{n} \quad \text{for } n \geq 1}$$

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the recursive formula: gives the next term by way of the previous term (or terms)

example:
$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 & \text{for } n \geq 2 \end{cases}$$

give the first three terms

answer:
$$a_1 = 2$$
$$a_2 = a_1 + 3 = 2 + 3 = 5$$
$$a_3 = a_2 + 3 = 5 + 3 = 8$$

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example: Fibonacci sequence

$1, 1, 2, 3, 5, 8, 13, \ldots$

what is the recursive formula for this sequence?

answer:
$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} & \text{for } n \geq 3 \end{cases}$$

digression: will not be tested

what is the general formula for the Fibonacci sequence?

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

series: the sum of a sequence

examples:

a) $2 + 5 + 8 + \ldots$

b) $5 + 15 + 25 + \ldots + 105$

the sum of a finite sequence (finite series) is just a number

c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

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notation:

S_k = sum of the first k terms of a series

(if the series is finite, could be the sum of all terms)

note: also known as the

k^{th} partial sum

S_∞ = sum of all terms in an infinite series

note: if k is large, calculating S_k could be annoying! but we'll learn more efficient methods later

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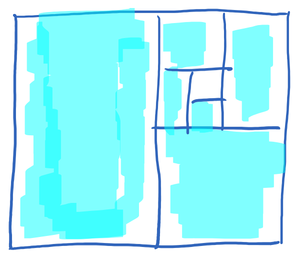
here's an interesting example:

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$



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consider the following series:

$16 + 20 + 24 + \ldots$

for this series, calculate S_3 and S_5

answer: $S_3 = 16 + 20 + 24 = 60$

$S_5 = 16 + 20 + 24 + 28 + 32 = 120$

note: it does not matter what the starting index is when calculating S_k

$S_3 = a_1 + a_2 + a_3$ if starting index is 1

$= a_0 + a_1 + a_2$ if starting index is 0

$= a_{17} + a_{18} + a_{19}$ 17

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sigma notation:

$$\sum_{n=1}^{4} (3n+1) = (3\cdot1+1) + (3\cdot2+1) + (3\cdot3+1) + (3\cdot4+1)$$

$$= 4 + 7 + 10 + 13$$

$$= 34$$

$$\sum_{i=0}^{2} 3^i = 3^0 + 3^1 + 3^2$$

$$= 1 + 3 + 9$$

$$= 13$$

$$\sum_{j=2}^{8} 7 = 7 + 7 + 7 + 7 + 7 + 7 + 7$$

$$= 49$$

note: how many terms? rule is

$k = n - m + 1$

final index   starting index

write the following series in sigma notation

$$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \cdots$$

answer: $\displaystyle\sum_{n=6}^{\infty} \frac{1}{n}$

or $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k+5}$

or $\displaystyle\sum_{m=0}^{\infty} \frac{1}{m+6}$

digression: will not be tested

why do we care?

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$