Section 3.2: Arithmetic Sequences and Series

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examples:

	pattern?
a) 2, 5, 8, ...	add 3
b) 0.4, 0.5, 0.6, ...	add 0.1
c) 5, −5, −15, ... −205	add (−10)
d) −1, $-\frac{3}{2}$, −2, $-\frac{5}{2}$, ... −10	add $(-\frac{1}{2})$

arithmetic sequence ≡ a sequence in which you find the next term by adding a constant to the previous term

→ common difference d

how to find it? subtract any term from the next term to find it

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recursive form for arithmetic:

example: write the recursive form for the sequence

0.4, 0.5, 0.6, ...

answer:
$$\begin{cases} a_4 = 0.4 \\ a_n = a_{n-1} + 0.1 \end{cases}$$
for $n > 4$ or $n \geq 5$

in general,
$$\begin{cases} a_m = \text{<insert first term here>} \\ a_n = a_{n-1} + d \end{cases}$$
for $n > m$ or $n \geq m+1$

$\{ \; a_n = a_{n-1} + d$ for $n \geq \ldots$
or $n \geq m+1$

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general formula: $2, 5, 8, \ldots$

①	②	③	④		
$2,$	$5,$	$8,$	11	$\ldots$	a_n
$2,$	$2+3,$	$2+2\cdot 3,$	$2+3\cdot 3,$	$\ldots$	$2+(n-1)3$

so for this particular example,

$$a_n = 2 + (n-1)\cdot 3$$

$$\Rightarrow \boxed{a_n = a_1 + (n-1)\cdot d \quad \text{for} \quad n \geq 1}$$

and in general

$$\boxed{a_n = a_m + (n-m)\cdot d \quad \text{for} \quad n \geq m}$$

note: to find the <u>simplified</u> general formula for $2, 5, 8, \ldots$

we need to simplify:
$$\begin{aligned} a_n &= 2 + (n-1)\cdot 3 \\ &= 2 + 3n - 3 \\ &= 3n - 1 \quad \text{for } n \geq 1 \end{aligned}$$

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example: find a general formula for the sequence

$5, -5, -15, \ldots$

simplify your answer and specify your starting index

answer: this sequence is arithmetic
let's use $m=0$ as our starting index

$a_0 = 5$
$d = -10$

$a_n = a_m + (n-m)d$

$= 5 + (n-0)(-10)$

$\boxed{a_n = 5 - 10n \quad \text{for } n \geq 0}$

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the advantage of the general form is that calculating terms is more straightforward than for the recursive formula

- for general, to find a_{1000}, plug $n=1000$ into the a_n formula

- for recursive, to find a_{1000}, you need to first calculate a_{999}

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example: for the arithmetic sequence in which the first term is 2 and the fiftieth term is 394, what is the common difference?

answer: $a_n = a_m + (n-m)d$

let's use $n \geq 1$ $\quad a_n = a_1 + (n-1)d$

$a_1 = 2$
$a_{50} = 394$

$394 = 2 + (50-1) \cdot d$

$392 = 49 \cdot d$

$d = 8$

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example: Consider the arithmetic sequence which has a tenth term of 5 and a twenty-fifth term of -70. Give the recursive formula for this sequence.

$$a_n = a_m + (n-m)d$$

$a_{10} = 5$ $\quad$ $a_{25} = -70$

$$-70 = 5 + (25-10)\cdot d$$

$$-75 = 15d$$

$$d = -5$$

but what's the first term?

$a_1 = ?$
$a_{10} = 5$

$$a_n = a_m + (n-m)\cdot d$$

$$5 = a_1 + (10-1)(-5)$$

$$5 = a_1 - 45$$

$$a_1 = 50$$

recursive: $\begin{cases} a_1 = 50 \\ a_n = a_{n-1} - 5 \quad \text{for} \quad n \geq 2 \\ \phantom{a_n = a_{n-1} - 5 \quad \text{for} \quad} n > 1 \end{cases}$

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arithmetic series:

$$2 + 5 + 8 + \ldots$$

for this series, calculate S_8

answer: $S_8 = \overbrace{2 + \underbrace{5 + 8 + \overbrace{11 + 14}^{} + 17 + 20}_{25} + 23}^{25}$

answer: $S_8 = \underbrace{2 + 5 + 8 + \underbrace{11 + 14}_{25} + 17 + 20 + 23}_{25}$

$= 4 \times 25$

number of pairs

sum of each pair

$S_k = \frac{k}{2}(a_m + a_n)$

a_m = first term
a_n = final term

$k = n - m + 1$

what if k is odd?

$S_7 = \underbrace{2 + \overbrace{5 + 8 + \underbrace{11 + 14}_{} + 17}^{22} + 20}_{22}$

the formula still works because we have $3\frac{1}{2}$ $\left(\frac{7}{2}\right)$ pairs that sum to 22

summary:

$$\boxed{\begin{array}{l}\text{for arithmetic} \\ S_k = \frac{k}{2}(a_m + a_n) \\ S_k = \frac{k}{2}\left(2a_m + (n-m)\cdot d\right)\end{array}}$$

$a_n = a_m + (n-m)\cdot d$

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example: find the sum of the first 50 terms of

$2 + 5 + 8 + \ldots$

method #1: arithmetic with $d = 3$

$m = 1$ and $a_1 = 2$

$k = 50$

$$S_k = \frac{k}{2}\left(2a_m + (n-m)\cdot d\right)$$

$$= \frac{50}{2}\left(2\cdot 2 + (50-1)\cdot 3\right)$$

$$= 3775$$

method #2: arithmetic with $d = 3$

$$S_k = \frac{k}{2}\left(a_m + a_n\right)$$

$$= \frac{50}{2}\left(2 + \quad\right)$$

but what is a_n?

$$a_n = a_m + (n-m)d$$

$$a_{50} = 2 + (50-1)\cdot 3$$

$$= 2 + 49\cdot 3$$

$$= 149$$

$$S_k = \frac{50}{2}\left(2 + 149\right)$$

$$= 3775$$

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evaluate:

$$\sum_{j=4}^{50} (6j-3) = \underset{(4)}{21} + \underset{(5)}{27} + \underset{(6)}{33} + \ldots + \underset{(50)}{297}$$

arithmetic with $d = 6$

$a_4 = 21$ so $m = 4$

$a_{50} = 297$ so $n = 50$

$k = n - m + 1 = 47$

$$S_k = \frac{k}{2}\left(a_m + a_n\right)$$

$$S_k = \frac{k}{2}\left(a_m + a_n\right)$$

$$= \frac{47}{2}\left(21 + 297\right)$$

$$= 7473$$