Section 3.3: Geometric Sequences and Series

Thursday, October 3, 2019 10:36 AM

examples:

pattern?

1. $7, 14, 28, 56, \ldots 114688$ — mult by 2
2. $100, 20, 4, 4/5, \ldots$ — mult by $1/5$
3. $1/2, 1/4, 1/8, 1/16, \ldots 1/256$ — mult by $1/2$
4. $24, -16, \frac{32}{3}, \frac{-64}{9}, \ldots$ — mult by $\frac{-2}{3}$

how to find this?
take any term (except the first) and divide by the previous term

so $\frac{-16}{24} = \frac{-2\cdot\cancel{8}}{3\cdot\cancel{8}} = -\frac{2}{3}$ (or $-0.\overline{6}$)

geometric sequence ≡ a sequence in which the next term is equal to the previous term multiplied by a constant

common ratio r

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recursive formula:

example: give a recursive formula for the sequence

$100, 20, 4, 4/5, \ldots$

geometric with $r = \frac{1}{5}$

$$\begin{cases} a_0 = \boxed{100} \\ a_n = \underbrace{\frac{1}{5}}_{\text{common ratio } r} a_{n-1} & \text{for } n > 0 \end{cases}$$

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general formula:

example: find a general formula for

$7, 14, 28, \ldots$

1	2	3		n
$7,$	$14,$	$28,$	$\ldots$	a_n
$7,$	$7 \cdot 2,$	$7 \cdot 2^2,$	$\ldots$	$7 \cdot 2^{n-1}$

for this sequence, it's $\boxed{a_n = 7 \cdot 2^{n-1} \quad \text{for } n \geq 1}$

for geometric, $\boxed{a_n = a_m r^{n-1} \quad \text{for } n \geq m}$

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example: consider the sequence

$5, 15, 45, \ldots$

find a) the twelfth term
b) the fiftieth term

answer: geometric with $r=3$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

if $m=1$, want a_{12} and a_{50}

$$a_n = 5 \cdot 3^{n-1}$$

$$a_{12} = 5 \cdot 3^{11} = 885\,735$$

$$a_{50} = 5 \cdot 3^{49} = 1.196 \times 10^{24}$$

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give the general formula for

$$80, \quad -160, \quad 320, \quad \ldots$$

answer: geometric with $r = -2$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

$$\boxed{a_n = 80(-2)^{n-1} \quad \text{for } n \geq 1}$$

brackets are necessary

note:

$80 * -2^{n-1}$ is not correct

why not?

$$-2^2 = -1 \cdot 2^2 = -4 \quad \text{BEDMAS}$$

$(-2)^2 = (-2)(-2) = 4$

how to fix?

80 (-2) ^ (n-1)

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geometric series: $7 + 14 + 28 + \ldots$

$$S_k = \frac{a_m(1 - r^k)}{1 - r}$$

where a_m is the first term and $k \geq 1$ and $r \neq 1$

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why? digression: will not be tested

$$S_k = a_1 + a_2 + a_3 + a_4 + \ldots a_k$$

add
$$\left\{ \begin{array}{l} S_k = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots a_1 r^{k-1} \\ -rS_k = -a_1 r - a_1 r^2 - a_1 r^3 - a_1 r^4 - a_1 r^k \end{array} \right.$$

$$S_k - rS_k = a_1 - a_1 r^k$$

$$S_k(1-r) = a_1(1-r^k)$$

$$S_k = \frac{a_1(1-r^k)}{1-r}$$

where a_1 = first term

$1-r$ term

example : for the series $2 + 10 + 50 + \ldots$

find the sum of the first

a) twelve terms
b) fifty terms

answer: geometric with $r = 5$

$$S_k = \frac{a_m(1-r^k)}{(1-r)}$$

$$= \frac{2(1-5^k)}{1-5}$$

so $S_{12} = \frac{2(1-5^{12})}{1-5} = 122\ 070\ 312$

or 1.22×10^8

for scientific notation, please give at least two decimal places

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what about $1/4 + 1/16 + 1/64 + \ldots$?

how do we make this work?

let's try $S_k = \frac{a_m(1-r^k)}{1-r}$

$$S_\infty = \frac{a_m(1-r^\infty)}{1-r}$$

but what is r^∞?

let's consider $\left(\frac{1}{4}\right)^n$

as $n \to \infty$, $\left(\frac{1}{4}\right)^n \to 0$

approaches

$\boxed{\text{but}}$ only for

$-1 < r < 1$

or $|r| < 1$

then $S_\infty = \frac{a_m(1-\cancel{r}^{\to 0})}{1-r}$

$$\boxed{S_\infty = \frac{a_m}{1-r}}$$ for $-1 < r < 1$

so, back to the original question

$1/4 + 1/16 + 1/64 + \ldots$

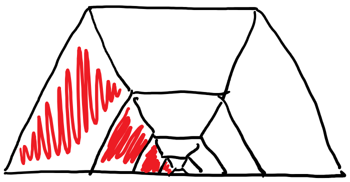
geometric with $r = 1/4$

is $-1 < r < 1$? yes ✓

$S_\infty = \frac{a_m}{}$

$$\frac{\quad}{1-r}$$

$$= \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$



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evaluate: $24 - 16 + \frac{32}{3} - \ldots$

geometric with $r = \frac{-16}{24} = \frac{-2\cdot 8}{3\cdot 8} = -\frac{2}{3}$

is $-1 < r < 1$? ✓

$$S_\infty = \frac{a_m}{1-r}$$

$$= \frac{24}{1+2/3} = \frac{24}{5/3} = 24\cdot\frac{3}{5} = \frac{72}{5}$$

or 14.4

evaluate

$12 + 18 + 27 + \ldots$

$$\cancel{S_\infty = \frac{a_m}{1-r} = \frac{12}{1-3/2} = \frac{12}{-1/2} = 12(-2) = -24}$$

is $-1 < r < 1$? NO

answer: S_∞ is undefined

or S_∞ does not exist (DNE)

note: for this particular example, you could also say

$S_\infty = \infty$ for $12 + 18 + 27 + \ldots$

but $S_\infty = -\infty$ for $-12 - 18 - 27 - \ldots$

and $S_\infty = \text{DNE}$ $12 - 18 + 27 - \ldots$

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evaluate:

$$\sum_{j=0}^{\infty} 75\left(\frac{3}{5}\right)^j = \overset{0}{75\left(\frac{3}{5}\right)^0} + \overset{1}{75\left(\frac{3}{5}\right)^1} + \overset{2}{75\left(\frac{3}{5}\right)^2} \ldots$$

$$= 75 + 75\left(\frac{3}{5}\right) + 75\left(\frac{3}{5}\right)^2 + \ldots$$

geometric with $a_m = 75$

$r = 3/5$

$-1 < r < 1$? ✓

$$S_\infty = \frac{a_m}{1-r} = \frac{75}{1-3/5} = \frac{75}{2/5}$$

$$= 187.5$$