Section 4.1: Big O and Rates of Growth

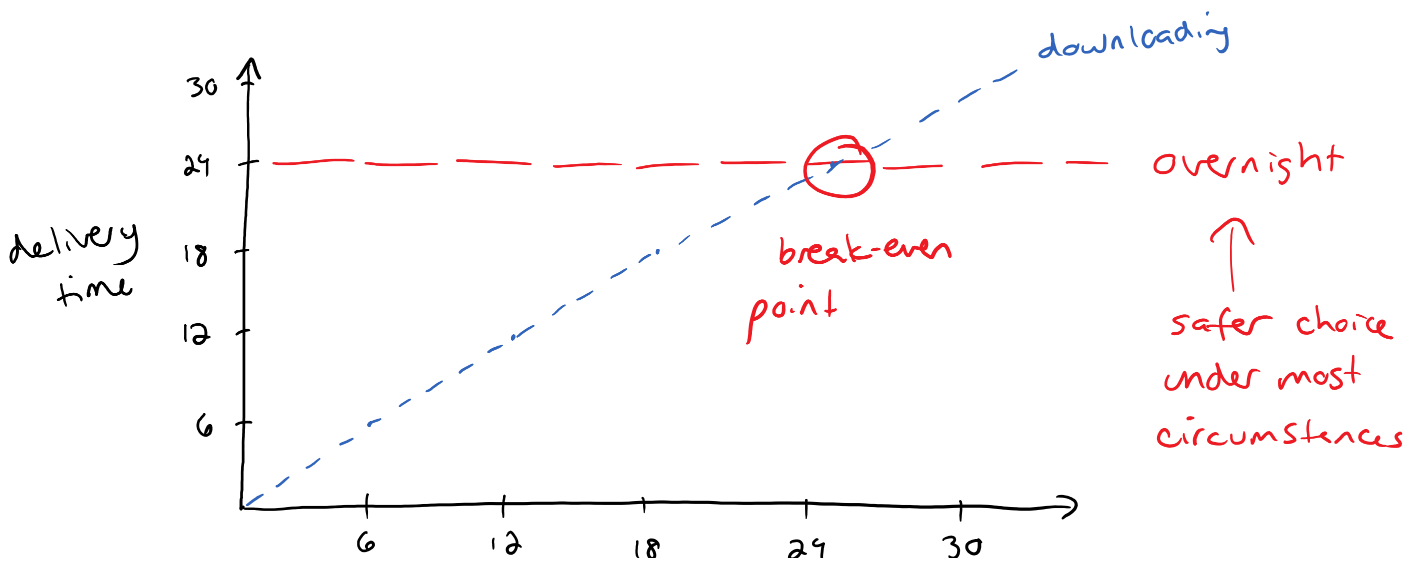
Friday, October 11, 2019 10:50 AM

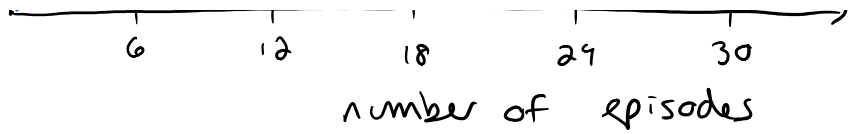
example: Suppose you wish to download a number of Star Trek episodes. You can either download them at one episode per hour, or you could buy them on DVD from Amazon with overnight delivery (let's call that 24 hour delivery).

scenarios: minimum delivery time
(1 hour / episode vs 24 hours total)

- few episodes, < 24, downloading faster
- break even, = 24, methods have the same time
- many episodes, > 24, overnight faster

and if you don't know how episodes will be required, the "many episodes" scenario is the safest approach





in computing, we are often trying to minimize the number of operations necessary to run an algorithm

let's look at the number of steps required for two algorithms

① consider adding two 5-digit numbers

```
  12345
+ 52423
```

best-case : 5 additions, one for each column

worst-case : 5 additions, plus some carries

② consider multiplying two 5-digit numbers

```
      12345
    × 52423
      xxxxx
     xxxxx
    xxxxx
   xxxxx
  xxxxx
```

best case : 25 multiplications plus 9 column additions

worst case : 25 multiplications plus 10 column additions plus lots of carries the worrying part is the fact that the number of multiplications goes as the <u>square</u> of the number of digits

100 digits → 10 000 multiplications! ouch!

Big O:

let n be the number of elements you are considering

(n is the number of digits in your number, or the number of Star Trek episodes)

- size of input

then O is the number of operations needed for the task

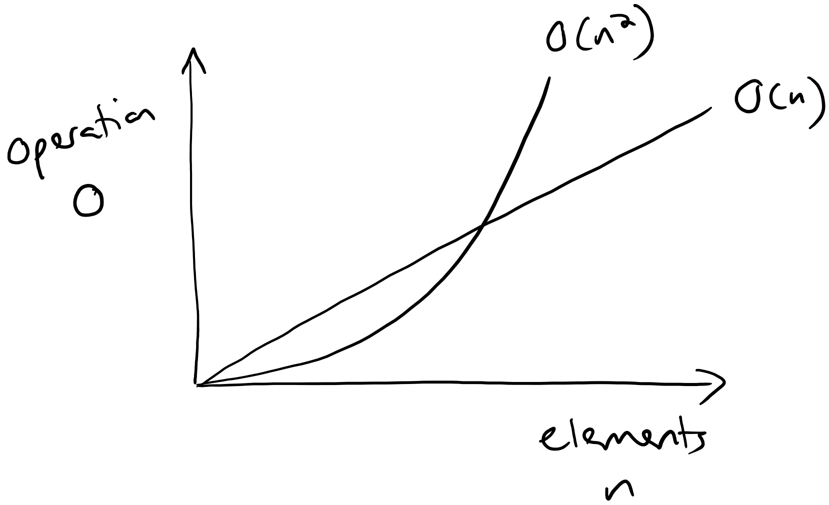
addition: if you double the number of digits you approximately double the number of operations

Big O notation: $O(n)$
order n

multiplication: if you double the number of digits you approximately quadruple the number of operations

$O(n^2)$

note: I will not ask you to analyze an algorithm and figure out Big O



so Big O only talks about the number of operations for large n

and in this graph, $O(n^2)$ will be larger than $O(n)$