MATH 155 – Practice Test 2

October 10, 2017 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 30 points

1. (4 points) The following statement is true: "If and only if you draw the winning number, you will win a prize." Given that, answer the following questions by circling the correct choice.

   ↑ so the biconditional

   (a) You don't draw the winning number. Do you win a prize? Yes / (No) / Maybe

   (b) You draw the winning number. Do you win a prize? (Yes) / No / Maybe

   (c) You win a prize. Did you draw the winning number? (Yes) / No / Maybe

   (d) You do not win a prize. Did you draw the winning number? Yes / (No) / Maybe

   $p \Leftrightarrow q$ means that p and q are either both true or both false

2. (4 points) Consider the conditional "If the medusa looks at you, you will be turned to stone." If this is true, can the following situations occur? Circle the correct choice.

   (a) The medusa looks at you and you are turned to stone. (Yes) / No

   (b) The medusa looks at you and you are not turned to stone. Yes / (No)

   (c) The medusa does not look at you and you are turned to stone. (Yes) / No

   (d) The medusa does not look at you and you are not turned to stone. (Yes) / No

   the only one that cannot occur is first one true and second one false

3. (2 points) For the conditional $p \to q$, the contrapositive is $\overline{q} \to \overline{p}$. Consider the conditional "If Pat is very hungry, she will eat a burger and yam fries." Which of the following is the corresponding contrapositive statement? (Circle one.)

   (a) If Pat ate a burger and yam fries, then she was very hungry.

   (b) If Pat did not eat a burger and did not eat yam fries, then she was not very hungry.

   (c) If Pat did not eat a burger or did not eat yam fries, then she was not very hungry.

   (d) If Pat ate a burger or yam fries, then she was very hungry.

   ↑ DeMorgan's!

Let p = "weebles wobble"

4. (3 points) Consider the following statements about weebles. (You do not need to know what a weeble is or whether it wobbles to answer this question.)

   (a) Weebles wobble and don't wobble. $p \wedge \bar{p} \iff 0$

   (b) Weebles wobble or don't wobble. $p \vee \bar{p} \iff 1$

   (c) Weebles wobble and they wobble. $p \wedge p \iff p$

   (d) Weebles wobble or they wobble. $p \vee p \iff p$

   For the questions below, write the letter (a, b, c, or d) of the statement(s) above that meets the given condition. You may choose more than one answer. If none of the statements fit, write "none" in the blank.

   - List any statements that are always true. b (1)
   - List any statements that are always false. a (1)
   - List any statements that are equivalent to "Weebles wobble." c, d (1)

5. (4 points) Use a truth table to simplify the logical expression $(p \to q) \wedge (\bar{p} \to q)$.

p	q	$\bar{p}$	$p \to q$	$\bar{p} \to q$	$(p \to q) \wedge (\bar{p} \to q)$
0	0	1	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	1	0	1	1	1

same

so $(p \to q) \wedge (\bar{p} \to q)$ simplifies to $\boxed{q}$

no conclusion (−1)

each error on table (−½)

major mistake each (−1)

6. (2 points) Calculate all terms of the following sequence and write your answer on the blank line.

$$a_n = 24 - 6n \qquad \text{for } 1 \le n \le 4$$

18, 12, 6, 0

$a_1 = 24 - 6(1) = 18$

$a_2 = 24 - 6(2) = 12$

$a_3 = 24 - 6(3) = 6$

$a_4 = 24 - 6(4) = 0$

7. (2 points) Write a recursive formula for the following sequence.

$$3, 6, 12, 24, \ldots$$

pattern is multiplying by 2

so

$$\begin{cases} a_0 = 3 \\ a_n = 2a_{n-1} \quad \text{for } n \ge 1 \end{cases} *$$

* can start counting at any integer value

8. (3 points) Evaluate the following.

$$\sum_{i=4}^{7} i^2 + 1$$

130

$= (4^2+1) + (5^2+1) + (6^2+1) + (7^2+1)$

$= 17 + 26 + 37 + 50$

$= 130$

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

9. (4 points) Prove that the following Boolean expressions are equivalent by using the laws of logic. If you're stuck, try using a truth table for part marks.

$$A + \overline{A + B} = A + \overline{B}\,C + \overline{B}\,\overline{C}$$

-1 each minor mistake
-2 each major mistake
-½ name of law missing / incorrect

$A + \bar{A}\,\bar{B}$ = " DeMorgan's
$A + \bar{B}$ = " absorption
$= A + \bar{B}\,(C + \bar{C})$ distributive
$= A + \bar{B}\,(1)$ complement
$= A + \bar{B}$ identity

✓

10. (2 points) Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression.

$$\overline{\overline{p} \vee \overline{q} \vee \overline{q} \vee p}$$

-½ if omitted brackets here but rest of steps okay
-1 if omitted brackets and immediately made an error

→

$(\bar{p} \vee \bar{q}) \wedge (\bar{q} \vee p)$ DeMorgan's
$\bar{q} \vee (\bar{p} \wedge p)$ distributive
$\bar{q} \vee 0$ complement
$\bar{q}$ identity