MATH 155 – Test 2

February 7, 2020 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 30 points

1. (3 points) The following statement is true: "If you eat at Joe's, then you will have a good meal." Given that, can the following situations occur? Circle the correct choice.

p

q

   (a) You did not eat at Joe's and you had a good meal. Yes / No

   (b) You did not eat at Joe's and you had a bad meal. Yes / No

   (c) You ate at Joe's and you had a bad meal. Yes / No

you can't have p true (eat at Joe's) and q false (not good meal)

2. (3 points) Consider the statement: "If and only if a quantity is conserved, then a symmetry is exhibited." Answer the following questions, circling the correct choice.

   (a) A quantity is not conserved. Is a symmetry exhibited? Yes / No / Maybe

   (b) A symmetry is exhibited. Is a quantity conserved? Yes / No / Maybe

   (c) A symmetry is not exhibited. Is a quantity conserved? Yes / No / Maybe

either p and q are both true or both false

3. (2 points) Consider the statement $p \to q$: "If you break a mirror, then you will have seven years of bad luck." Which of the following statements are logically equivalent to $p \to q$? Circle all of the correct answers.

   (a) If you don't break a mirror, you won't have seven years of bad luck.

   (b) If you do not have seven years of bad luck, then you did not break a mirror. ← contrapositive

   (c) If you have seven years of bad luck, then you broke a mirror.

   (d) Either you did not break a mirror or you had seven years of bad luck or both. ← "or" form

$\bar{p} \vee q$

p	q	$\bar{p}$	$p \to q$	$\bar{p} \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

same

4. (3 points) Consider the statement, "This apple is red." Which of the following are logically equivalent to that statement? Circle any correct answers. You may choose more than one.

   (a) It is not true that this apple is not red. $\bar{\bar{p}} \Leftrightarrow p$ ✓

   (b) This apple is red and this apple is red. $p \wedge p \Leftrightarrow p$ ✓

   (c) This apple is red or this apple is not red. $p \vee \bar{p} \Leftrightarrow 1$ ×

   (d) This apple is both red and shiny or this apple is red but not shiny.
   $(p \wedge q) \vee (p \wedge \bar{q})$
   $p \wedge (q \vee \bar{q})$
   $p \wedge 1$
   p ✓

   (e) This apple is red or this apple is both red and shiny. $p \vee (p \wedge q) \Leftrightarrow p$ ✓

   (f) This apple is red or this apple is not red but it is shiny. $p \vee (\bar{p} \wedge q) \Leftrightarrow p \vee q$ ×

(−1) for every correct one missed
(−1) for every incorrect one chosen

5. (4 points) Use a truth table to simplify the logical expression $(p \leftrightarrow \overline{q}) \wedge (p \leftrightarrow q)$.

p	q	$\bar{q}$	$p \leftrightarrow \bar{q}$	$p \leftrightarrow q$	$(p \leftrightarrow \bar{q}) \wedge (p \leftrightarrow q)$
0	0	1	0	1	0
0	1	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	0

Conclusion: $\boxed{0}$

6. (5 points) Consider the sequence given by the following.

$$\begin{cases} a_0 = 4 \\ a_n = 3a_{n-1} & \text{for } n \geq 1 \end{cases}$$

(a) Is this formula recursive or general? Circle one: recursive / general

(b) Is it finite or infinite? Circle one: finite / infinite

(c) Calculate the first three terms of this sequence: $\underset{a_0}{\underline{4}}$, $\underset{a_1}{\underline{12}}$, $\underset{a_2}{\underline{36}}$

$a_1 = 3a_0 = 12$

$a_2 = 3a_1 = 36$

7. (4 points) Evaluate the following. Show your work.

$$\sum_{k=3}^{8}(2k-5)$$

$\underline{36}$

$$\overset{3}{=(2\cdot 3-5)} + \overset{4}{(2\cdot 4-5)} + \overset{5}{(2\cdot 5-5)} + \overset{6}{(2\cdot 6-5)} + \overset{7}{(2\cdot 7-5)} + \overset{8}{(2\cdot 8-5)}$$

$$= 1 + 3 + 5 + 7 + 9 + 11$$

$$= 36$$

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

8. (4 points) Prove the following using the laws of logic. If you're stuck, try using a truth table for part marks.

$$A + (\overline{C} + 0)(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$$

$A + \overline{C}(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

$A + \overline{C} \cdot 1 = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ complement

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A}}$ idempotent

$= \overline{A}\,\overline{C} + A$ complement

$= \overline{C} + A$ absorption

9. (2 points) Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression.

$$\left(\overline{A}B + \overline{A + B}\right)\,\overline{\overline{A}B}$$

$\overline{A+B}\;\overline{\overline{A}B}$ absorption

$\overline{A + B + \overline{A}B}$ De Morgan's

either is fine

$\overline{A+B}$ absorption

$\overline{A}\,\overline{B}$ De Morgan's