

# MATH 155 – Test 2: Version A

October 8, 2019

Name: Solution Set

Instructor: Patricia Wrean

Total: 30 points

1. (2 points) The following statement is true: “If you forget your password, then you will not be able to log on.” Given that, answer the following questions by circling the correct choice.

(a) You forgot your password. Will you be able to log on? Yes /  No / Maybe

(b) You were able to log on. Did you forget your password? Yes /  No / Maybe

(c) You were not able to log on. Did you forget your password? Yes / No /  Maybe

(d) You did not forget your password. Will you be able to log on? Yes / No /  Maybe

2. (2 points) The following statement is true: “If and only if you lose a library book, then you will have to pay for a replacement.” Given that, is it possible for each of the following statements to be true? Circle the correct choice.

(a) You lose a library book and pay for a replacement.  Yes / No

(b) You do not lose a library book but you pay for a replacement. Yes /  No

(c) You lose a library book but do not pay for a replacement. Yes /  No

(d) You do not lose a library book and you do not pay for a replacement.  Yes / No

3. (2 points) Consider the statement: “If everything is awesome, then you are living the dream.” Which of the following statements are logically equivalent? Circle all of the correct answers.

(a) If everything is not awesome, then you are not living the dream.

(b) If you are living the dream, then everything is awesome.

(c) If you are not living the dream, then everything is not awesome.

(d) Either you are living the dream and everything is awesome or you are not living the dream and everything is not awesome.

$$\left. \begin{array}{l} p \rightarrow q \\ \bar{q} \rightarrow \bar{p} \end{array} \right\} \text{logically equivalent}$$

2 if correct answer circled  
 -1 for every other answer circled

4. (4 points) Consider the following statement: "This cup of tea contains  $p \vee q$  milk or sugar." Circle all statements below that are logically equivalent. There may be more than one correct answer.

- (a) It is not true that this cup of tea does not contain milk or does not contain sugar.  
 (b) It is not true that this cup of tea does not contain milk and does not contain sugar.  
 (c) This cup of tea contains milk or it contains sugar but not milk.  
 (d) This cup of tea contains milk or it contains both milk and sugar.

a)  $\overline{\bar{p} \vee \bar{q}} \Leftrightarrow p \wedge q$  by DeMorgan's, so no  
 b)  $\overline{\bar{p} \wedge \bar{q}} \Leftrightarrow p \vee q$  " so yes  
 c)  $p \vee (q \wedge \bar{p}) \Leftrightarrow p \vee q$  by absorption, so yes  
 d)  $p \vee (p \wedge q) \Leftrightarrow p$  " so no

5. (4 points) Is  $(\bar{p} \vee q) \rightarrow (\bar{p} \wedge q)$  logically equivalent to  $p \leftrightarrow \bar{q}$ ? Use a truth table to justify your answer.

$p$	$q$	$\bar{p}$	$\bar{q}$	$\bar{p} \vee q$	$\bar{p} \wedge q$	$(\bar{p} \vee q) \rightarrow (\bar{p} \wedge q)$	$p \leftrightarrow \bar{q}$
0	0	1	1	1	0	0	0
0	1	1	0	1	1	1	1
1	0	0	1	0	0	1	1
1	1	0	0	1	0	0	0

Yes

- (-1) no conclusion  
 (- $\frac{1}{2}$ ) each incorrect value for max (-1) per column

6. (2 points) Calculate all terms for the following formula and write your answer on the line.

$$a_n = 4^n - 1 \quad \text{for } 0 \leq n \leq 2$$

0, 3, 15

$$a_0 = 4^0 - 1 = 1 - 1 = 0$$

$$a_1 = 4^1 - 1 = 4 - 1 = 3$$

$$a_2 = 4^2 - 1 = 15$$

(-1) each mistake

7. (3 points) Write a recursive formula for the following. Draw a box around your answer.

5, 11, 17, 23, ...

pattern: add 6

$$\begin{cases} a_0 = 5 \\ a_n = a_{n-1} + 6 \quad \text{for } n \geq 1 \end{cases}$$

(-1/2) no box

8. (5 points) Consider the following.

$$\frac{7}{8} + \frac{7}{9} + \frac{7}{10} + \dots + \frac{7}{25}$$

- (a) Is this a sequence or a series? Circle one:

sequence / series

(1)

- (b) Is it finite or infinite? Circle one:

finite / infinite

(1)

- (c) How many terms does it have?

$$k = n - m + 1 \\ = 25 - 8 + 1 = 18$$

18

(1)

- (d) Rewrite it using sigma notation. Don't bother calculating the total.

(2)

$$\sum_{j=8}^{25} \frac{7}{j}$$

$$\left( \text{or } \sum_{i=1}^{18} \frac{7}{i+7} \quad \text{or } \sum_{k=0}^{17} \frac{7}{k+8} \right)$$

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

9. (4 points) Simplify the following using the laws of logic. If you're stuck, try using a truth table for part marks.

$$\begin{aligned}
 & (\bar{p} \wedge (q \vee 0)) \vee (\bar{p} \wedge (\bar{q} \vee \bar{q})) \\
 & (\bar{p} \wedge q) \vee (\bar{p} \wedge (\bar{q} \vee \bar{q})) && \text{identity} \\
 & (\bar{p} \wedge q) \vee (\bar{p} \wedge \bar{q}) && \text{idempotent} \\
 & \bar{p} \wedge (q \vee \bar{q}) && \text{distributive} \\
 & \bar{p} \wedge 1 && \text{complement} \\
 & \bar{p} && \text{identity}
 \end{aligned}$$

-1 each mistake  
 -1/2 missing/incorrect name of law

10. (2 points) Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression.

$$\begin{aligned}
 & \overline{\overline{A} \overline{C} + \overline{C}} (\overline{A \overline{B} + \overline{C} + \overline{A} \overline{C}}) \\
 & \overline{\overline{A} \overline{C} + \overline{C}} && \text{absorption} \\
 & \overline{\overline{C}} && \text{absorption} \\
 & C && \text{complement}
 \end{aligned}$$