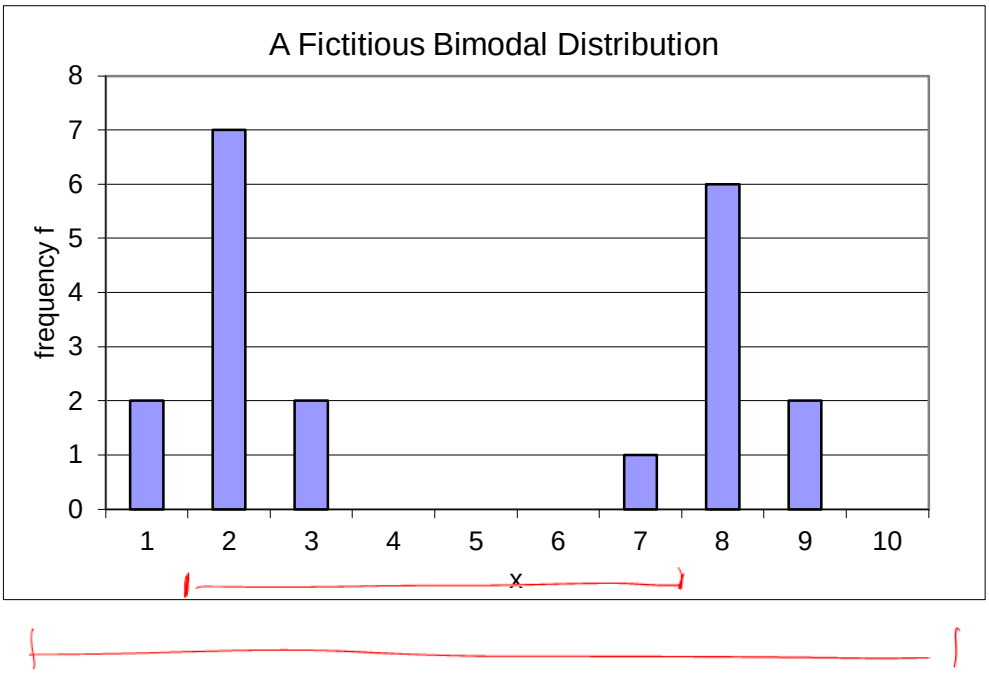
Math 156: Tchebysheff & Empirical Rules

Consider the following sample data set:

1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 7, 8, 8, 8, 8, 8, 8, 9, 9

The mean of this data set is 4.75 with standard deviation of 3.18. It has the following frequency histogram.



Complete the table below by finding the percentage of measurements in the intervals $\bar{x} \pm s$, $\bar{x} \pm 2s$ and $\bar{x} \pm 3s$. Also, state the percentages you'd expect to find in each interval according to the Empirical Rule and Tchebysheff's Theorem.

$\bar{x} = 4.75$

$s = 3.18$

approximately

	interval	# of points	% of points	Empirical	Tcheby	Empirical works?	Tcheby works?
$\bar{x} \pm s$	1.57 to 7.93	10	50%	~68%	—	no	—
$\bar{x} \pm 2s$	-1.61 to 11.11	all	100%	95%	≥ 75%	sort of	yes
$\bar{x} \pm 3s$	-4.79 to 14.29	all	100%	~99.7%	≥ 89%	yes	yes

Should the actual percentages agree with the Empirical Rule? With Tchebysheff?

Empirical : no, the data set is not unimodal

Tcheby : yes, it works for all data sets

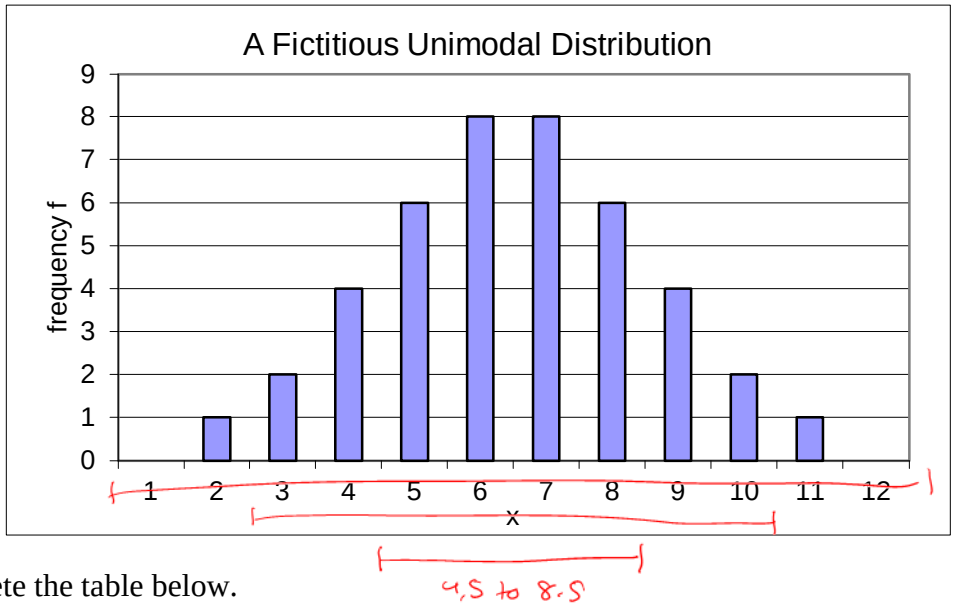
Consider another sample data set in which x is the value of the data point and f is the frequency with which that value occurs.

x	f
1	0
2	1
3	2
4	4
5	6
6	8
7	8
8	6
9	4
10	2
11	1
12	0

4.5 to 8.5

$6+8+8+6=28$

The mean of this data set is 6.5 with standard deviation 2.0. It has the following histogram.



Complete the table below.

$\bar{x} = 6.5$

$s = 2.0$

	interval	# of points	% of points	Empirical	Tcheby	Empirical works?	Tcheby works?
$\bar{x} \pm s$	4.5 to 8.5	28	$66.\overline{6}\%$	~68%	—	yes	—
$\bar{x} \pm 2s$	2.5 to 10.5	40	95.2%	~95%	$\geq 75\%$	yes	yes
$\bar{x} \pm 3s$	0.5 to 12.5	all	100%	~99.7%	$\geq 89\%$	yes	yes

Should the actual percentages agree with the Empirical Rule? With Tchebysheff?

Empirical: yes, because it is unimodal and symmetrical

Tcheby: yes, it always works

$\geq \left(1 - \frac{1}{k^2}\right)$ where $k > 1$