Section 1.4: Converting from Decimal

Monday, January 15, 2024 10:25 AM

we've converted from other bases to decimal — how do we go the other way?

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background: whole numbers: 0, 1, 2, 3, ...

integers: ..., -3, -2, -1, 0, 1, 2, 3, ...

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modular arithmetic

$\frac{7}{4} = 1\frac{3}{4}$ or 1.75

but if we stay with integers

$\frac{7}{4}$ has a quotient of 1 plus 3 left over

whole number or integer part

quotient Q

remainder R

how do you find Q and R using a calculator?

$\frac{7}{4} = 1.75$ on calculator

integer part 1

non-integer part 0.75

0.75

Q → 1

remainder $R = 0.75 \times 4 = 3$

note: how do you re-enter the non-integer part quickly?
subtract the integer part

examples: find Q and R for

a) $50 \div 4 = 12.5$

b) $92 \div 8 = 11.5$

c) $133 \div 16 = 8.3125$

$Q = 12$
$R = 0.5 \times 4 = 2$

$Q = 11$
$R = 0.5 \times 8 = 4$

$Q = 8$
$R = 0.3125 \times 16 = 5$

application: unit conversion

time: 45 days

= 6 weeks (Q) and 3 days (R)

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digression: will not be tested in this class

in computing, the modulus (or modulo) function computes the remainder left after dividing one integer by another

$\text{mod}(5, 3) = 2$
$5 \text{ mod } 3 = 2$
$5 \% 3 = 2$

recall: converting from octal to decimal

$$152_8 = 1\times8^2 + 5\times8^1 + 2\times8^0 = 106_{10}$$

how to go the other way?

repeated division

Convert 106_{10} to octal

	Q	R
106 ÷ 8	13	0.25 × 8 = 2
13 ÷ 8	1	0.625 × 8 = 5
1 ÷ 8	0	1

↑ keep dividing until Q = 0

↑ write these digits in reverse order

answer: 152_8

convert 58 to binary:

	Q	R
58 ÷ 2	29	0
29 ÷ 2	14	0.5 × 2 = 1
14 ÷ 2	7	0
7 ÷ 2	3	1
3 ÷ 2	1	1
1 ÷ 2	0	1

111010_2

convert 53710 to hexadecimal:

	Q	R	
$53710 \div 16$	3356	~~14~~	E
$3356 \div 16$	209	~~12~~	C
$209 \div 16$	13	1	
$13 \div 16$	0	~~13~~	D

$DICE_{16}$

convert 547 to base 5:

	Q	R
$547 \div 5$	109	2
$109 \div 5$	21	4
$21 \div 5$	4	1
$4 \div 5$	0	4

4142_5

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what about non-integer numbers?

instead of dividing by the base repeatedly, we multiply

so to convert 0.375_{10} to binary

→ multiply by the base and split it into the integer part and the non-integer part

		integer		non-integer
0.375 × 2 =		0	+	0.75
0.75 × 2 =		1	+	0.5
0.5 × 2 =		1	+	0

↑ once this is zero, you can stop

now take the integer part

0
1
1 ↓

$0.375_{10} = 0.011_2$

Section 1.4: cont'd 2024/01/16

example: convert 0.734375 to hexadecimal

		int		non-int
0.734375 × 16 =		~~11~~ B	+	0.75
0.75 × 16 =	↓	~~12~~ C	+	0

$0.BC_{16}$

example: convert to binary: 0.8125

		int		non-int
0.8125 × 2 =		1	+	0.625
0.625 × 2 =		1	+	0.25
0.25 × 2 =		0	+	0.5
0.5 × 2 =		1	+	0

$0.25 \times 2 = 0 + 0.5$

$0.5 \times 2 = 1 + 0$

0.01_2

an example with a twist:

convert 0.1 to binary

		int		non-int
0.1	× 2 =	0	+	0.2
0.2	× 2 =	0	+	0.4
0.4	× 2 =	0	+	0.8
0.8	× 2 =	1	+	0.6
0.6	× 2 =	1	+	0.2
0.2	× 2 =	0	+	0.4
0.4	× 2 =	0	+	0.8
0.8	× 2 =	1	+	0.6
0.6	× 2 =	1	+	0.2

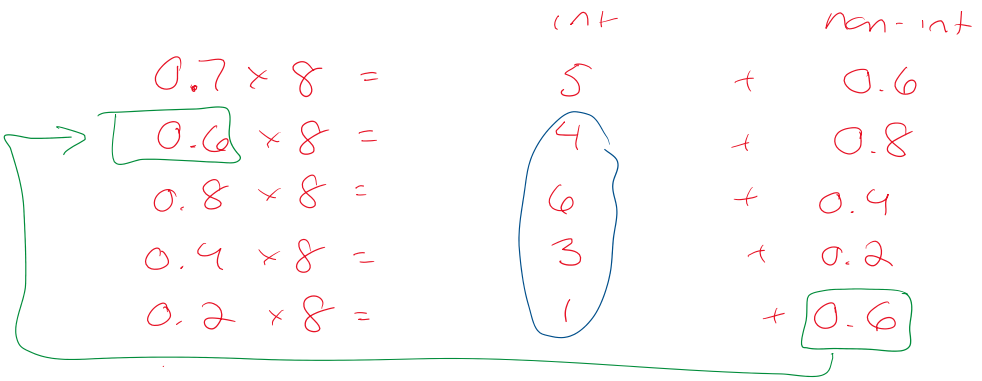
this is going to be a repeating pattern to the right of the radix point since the non-integer part will never go to zero

$0.1_{10} = 0.0\,0011\;0011\;0011\;0011\ldots$

$= 0.0\,\overline{0011}_2$

convert 0.7 to octal:

	int		non-int
$0.7 \times 8 =$	5	+	0.6



$0.7_{10} = 0.5\overline{4631}_8$

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putting it all together:

don't round

convert 19.96875 to octal. Give an exact answer.

integer part

	Q	R
19 ÷ 8	2	3
2 ÷ 8	0	2

non-integer part

	int	non-int
0.96875 × 8 =	7	+ 0.75
0.75 × 8 =	6	+ 0

answer: 23.76_8