Section 10.2: Calculating Large Sample Confidence Intervals for the Mean

Thursday, April 04, 2024 2:04 PM

when we talk about a confidence interval,

"The average length of adult Chinook salmon was found to be between 65 and 85 cm with 95% confidence"

what we mean is that we've calculated this interval (65 to 85 cm) with a method that predicts the actual result 95% of the time

in other words, 95 times out of 100 the true value will lie within 65 to 85 cm

<u>and</u> 5 times out of 100, the interval we've calculated will not contain the true value

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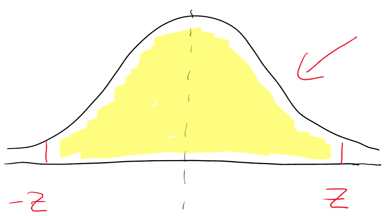
the most common confidence interval is a 95% confidence interval



the shaded region should have area equal to 0.95

so, what value of z makes this so?

○ Area from a value (Use to compute p from Z)
◉ Value from an area (Use to compute Z for confidence intervals) ← use this option

Specify Parameters:
Area 0.95 ← input area
Mean 0
SD 1
} leave as the default

Results:
Recalculate
○ Above
○ Below
◉ Between -1.96 and 1.96 ← choose this option
○ Outside

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the online calculator gives $z = 1.96$ for 95 % confidence

note: not exactly 2 but very close to 2 (Empirical Rule)

Section 10.2:

other common intervals are 90 %, 98 %, and 99 % and they also have z-scores associated with them

confidence level	z-score
90%	1.645
95%	1.96 *
98%	2.33
99%	2.575

* don't just use 2 from

* don't just use 2 from the Empirical Rule because 1.96 is more precise

for any other confidence interval, use the online calculator to find z

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to construct a confidence interval, you need to know the following pieces of information:

- confidence level (like 95%)
- the sample mean $\bar{x}$
- the standard deviation σ (for large sample, like we're doing here, can use sample standard deviation s instead)
- sample size n

then the population mean μ can be estimated by

$$\mu = \bar{x} \pm \underbrace{\frac{z\sigma}{\sqrt{n}}}_{\text{this whole term is called the margin of error}}$$

if σ is unknown, can use s for large samples margin of error (MOE)

note: this technique we are using only works for "large" samples in which

$n \geq 30$

example: Forty students were asked how much time they studied the weekend before final exams. The mean was found to be 15.1 hours with a standard deviation of 6.5 hours.

Construct confidence intervals for the mean time studied with

a) 90% confidence
b) 95% confidence
c) 99% confidence

What happens to the size of interval as the confidence level increases?

answer:

a) 90% confidence so $z = 1.645$

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

$$= 15.1 \pm \frac{1.645(6.5)}{\sqrt{40}}$$

$= 15.1 \pm \underbrace{1.69063}$

too many decimal places

in Math 156, round to the same number of decimal places as the mean

- or one more place

$= 15.1 \pm 1.7$

CI: 13.4 to 16.8 hours

b) same calculation but with 95% $\rightarrow z = 1.96$

$\mu = 15.1 \pm 2.01437$

$= 15.1 \pm 2.0$

CI: 13.1 to 17.1 hours

c) 99% conf $\rightarrow z = 2.575$

$\mu = 15.1 \pm 2.64643$

$= 15.1 \pm 2.6$

CI: 12.5 to 17.7 hours

As the confidence level increases, the width of the confidence interval also increases.

Section 10.2: cont'd 2024/09/09

what does increasing the sample size do?

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

margin of error
MOE

because n is in the denominator, increasing n decreases the MOE and thus decreases the width of the interval

drawing conclusions based on confidence intervals:

example: A study conducted by the doctors at a particular hospital involved monitoring a random sample of 75 surgery patients. The results showed that it took on average 3.2 mL of anesthetic A to put a patient to sleep, with a standard deviation of 0.4 mL.

However, the latest medical research indicates that the average amount of anesthetic A needed is 3.0 mL.

a) Calculate a 99% confidence interval for the amount of anesthetic A needed in this

a) Calculate a 99% confidence interval for the amount of anesthetic A needed in this hospital.

99% conf:

$z = 2.575$

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

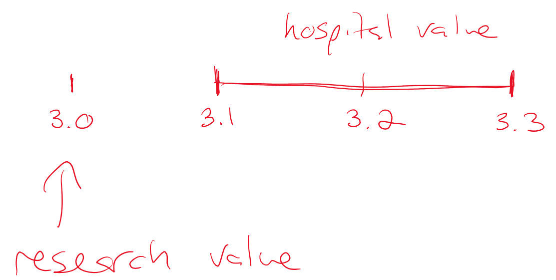
$$= 3.2 \pm \frac{2.575\,(0.4)}{\sqrt{75}}$$

$$= 3.2 \pm 0.1189$$

$$= 3.2 \pm 0.1$$

CI: 3.1 to 3.3 mL

b) Is there reason to believe that the hospital's value is different than the research value? Explain briefly.

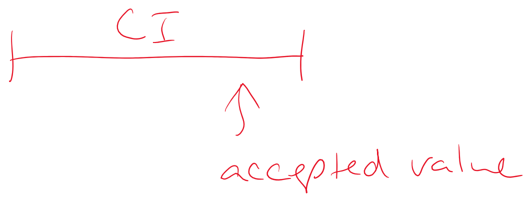


yes, because the research value lies <u>outside</u> the confidence interval

rule: comparing confidence interval to an accepted value,

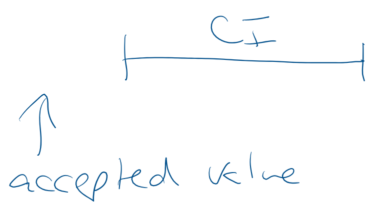
if accepted value is within interval

$\therefore$ in agreement



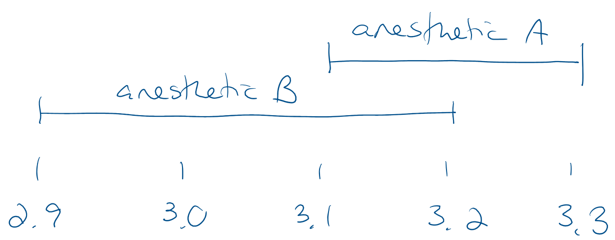
∴ in agreement

if accepted value is outside the interval



∴ disagreement

c) A similar study shows that the confidence interval for the amount of anesthetic B is 2.9 to 3.2 mL. Is there reason to believe that the amounts of A and B needed at this hospital are different? Explain briefly.



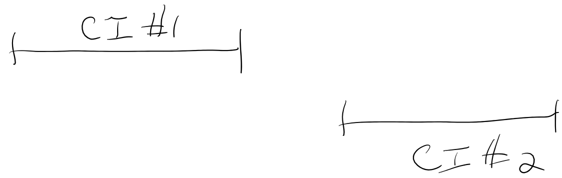
No, these amounts are not different. The intervals overlap.

rule: comparing two confidence intervals

if they overlap:

∴ agreement

if they do not overlap



∴ disagreement

how can you decrease the margin of error?

1. increase the sample size

   (good scientific approach)

2. decrease the confidence level

   (sad but sometimes practical approach)