Section 2.3: Logical Equivalence

Thursday, January 16, 2020 2:55 PM

consider the propositions p and q:

there are four possible combinations of values for p and q because each of them can be either true or false

truth tables: (extremely long version) (DON'T USE THIS)

p	q	$p \wedge q$
false	false	false
false	true	false
true	false	false
true	true	true

(or use F and T for false/true)

really short version:

let 0 = false
1 = true

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

truth tables:

truth tables:

p	q	$p \wedge q$	$p \vee q$	$\sim p$	$\sim(p \vee q)$
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	0	0

example: write the truth table for $p \oplus q$

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

example: write the truth table for $\sim(p \vee \sim q) \wedge \sim r$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim(p \vee \sim q) \wedge \sim r$
0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

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example: write the truth table for $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

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from the above table, we can see that

$$p \wedge \sim p \Leftrightarrow 0$$

"is logically equivalent to"

so we can use truth tables to simplify logical expressions

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example: use a truth table to simplify $p \wedge 1$

p	1	$p \wedge 1$
0	1	0
1	1	1

same

conclusion:

$$\boxed{p \wedge 1 \iff p}$$

simplify $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge \sim q$	$(\sim p \wedge \sim q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

$$\boxed{\sim q}$$

or $\boxed{(\sim p \wedge \sim q) \vee (p \wedge \sim q) \iff \sim q}$

Is $\sim(p \oplus q)$ logically equivalent to $\sim p \oplus \sim q$?
Use a truth table to justify your reasoning.

p	q	$\sim p$	$\sim q$	$p \oplus q$	$\sim(p \oplus q)$	$\sim p \oplus \sim q$
0	0	1	1	0	1	0
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

$\boxed{\text{NO}}$

or $\boxed{\sim(p \oplus q) \not\Leftrightarrow \sim p \oplus \sim q}$