Section 2.3: Logical Equivalence

Wednesday, January 24, 2024 11:37 AM

consider the propositions p and q

there are four possible combinations of the values of p and q because each can be either true or false

truth table (very long version)

p	q	$p \wedge q$
false	false	false
false	true	false
true	false	false
true	true	true

you could also use f/t or F/T if you wanted

really short version we will use

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

let 0 = false
1 = true

another truth table

p	q	$p \wedge q$	$p \vee q$	$\sim p$	$\sim(p \vee q)$	$p \oplus q$
0	0	0	0	1	1	0
0	1	0	1	1	0	1
1	0	0	1	0	0	1
1	1	1	1	0	0	0

example: write the truth table for $\sim(p \vee \sim q) \wedge \sim r$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim(p \vee \sim q) \wedge \sim r$
0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

example: write the truth table for $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

and from this table, we can see that

$$p \wedge \sim p \Leftrightarrow 0$$

"is logically equivalent to"

so we can use truth tables to simplify logical expressions

example: use a truth table to simplify $p \wedge 1$

p	1	$p \wedge 1$
0	1	0
1	1	1

0	1	0
1	1	1

same

conclusion: $p \wedge 1 \Leftrightarrow p$

$\boxed{p}$

simplify $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$ using a truth table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge \sim q$	$(\sim p \wedge \sim q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

same

$\boxed{\sim q}$

if you insist, $(\sim p \wedge \sim q) \vee (p \wedge \sim q) \Leftrightarrow \sim q$

Section 2.3: cont'd

example: Is $\sim(p \oplus q)$ logically equivalent to $\sim p \oplus \sim q$? Show your reasoning using a truth table.

p	q	$\sim p$	$\sim q$	$p \oplus q$	$\sim(p \oplus q)$	$\sim p \oplus \sim q$
0	0	1	1	0	1	0
0	1	1	0	1	0	1

0	0	1	1	0	1	0
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

$\boxed{\text{NO}}$

or, if you insist, $\sim(p \oplus q) \not\Leftrightarrow \sim p \oplus \sim q$