

# Section 2.3: Logical Equivalence

Wednesday, January 24, 2024 11:37 AM

consider the propositions  $p$  and  $q$

there are four possible combinations of the values of  $p$  and  $q$  because each can be either true or false

truth table (very long version)

$p$	$q$	$p \wedge q$
false	false	false
false	true	false
true	false	false
true	true	true

you could also use  $F/E$  or  $F/T$  if you wanted

really short version we will use

$p$	$q$	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

let 0 = false  
1 = true

another truth table

$p$	$q$	$p \wedge q$	$p \vee q$	$\sim p$	$\sim(p \vee q)$	$p \oplus q$
0	0	0	0	1	1	0
0	1	0	1	1	0	1
1	0	0	1	0	0	1
1	1	1	1	0	0	0

example: write the truth table for  $\sim(p \vee \sim q) \wedge \sim r$

$p$	$q$	$r$	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim(p \vee \sim q) \wedge \sim r$
0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

example: write the truth table for  $p \wedge \sim p$

$p$	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

and from this table, we can see that

$$p \wedge \sim p \iff 0$$

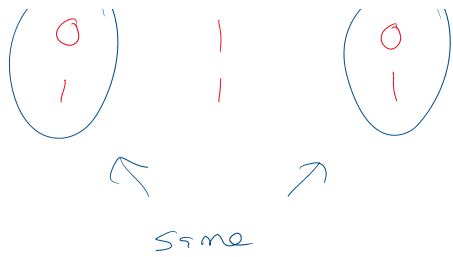
↑

"is logically equivalent to"

so we can use truth tables to simplify logical expressions

example: use a truth table to simplify  $p \wedge 1$

$p$	$1$	$p \wedge 1$
0	1	0
1	1	1



conclusion:  $p \wedge 1 \Leftrightarrow p$

$p$

simplify  $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$  using a truth table

$p$	$q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge \sim q$	$(\sim p \wedge \sim q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

same

$\sim q$

if you insist,  $(\sim p \wedge \sim q) \vee (p \wedge \sim q) \Leftrightarrow \sim q$

Section 2.3: cont'd

example: Is  $\sim(p \oplus q)$  logically equivalent to  $\sim p \oplus \sim q$ ? Show your reasoning using a truth table.

$p$	$q$	$\sim p$	$\sim q$	$p \oplus q$	$\sim(p \oplus q)$	$\sim p \oplus \sim q$
0	0	1	1	0	1	0
0	1	1	0	1	0	1

0	0	1	1	0		
0	1	1	0	1	1	0
1	0	0	1	1		
1	1	0	0	0		

NO

or, if you insist,  $\sim(p \oplus q) \not\equiv \sim p \oplus \sim q$