Section 2.5: The Laws of Logic (LOL)

Tuesday, January 30, 2024 10:49 AM

there are connections between logic and Boolean algebra

| logic | $p \wedge q$ | $p \vee q$ | $\sim p$ | F false | T true |
|---|---|---|---|---|---|
| Boolean algebra | $AB$ | $A+B$ | $\overline{A}$ | 0 | 1 |

we can show that

$p \wedge 1 \Leftrightarrow p$

can prove using a truth table

| $p$ | $1$ | $p \wedge 1$ |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

similarly, $A \cdot 1 = A$

these statements are true for all possible variables, so we call them laws. In particular, this one is called an identity law.

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identity laws: there are four of them:

$$p \wedge 1 \Leftrightarrow p$$
$$p \vee 1 \Leftrightarrow 1$$
$$p \wedge 0 \Leftrightarrow 0$$
$$p \vee 0 \Leftrightarrow p$$

but note that these statements are true for all possible varibles:

since $p \vee 0 \Leftrightarrow p$, then

$$q \vee 0 \Leftrightarrow q$$
$$\sim r \vee 0 \Leftrightarrow \sim r$$
☺ $\vee\, 0 \Leftrightarrow$ ☺
$$\sim(p \wedge q) \vee 0 \Leftrightarrow \sim(p \wedge q)$$

why do we care?

example: simplify the following using the laws of logic

- use only one law of logic per line
- state the name of the law you are using

$$(p \wedge 0) \vee (q \wedge 1)$$

$$0 \vee q \qquad \text{identity}$$

$$q \qquad \text{"}$$

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idempotent

$$p \wedge p \Leftrightarrow p$$

$$p \vee p \Leftrightarrow p$$

which means that $\sim r \wedge \sim r \Leftrightarrow \sim r$

$$\sim(p \wedge r) \vee \sim(p \wedge r) \Leftrightarrow \sim(p \wedge r)$$

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complement

$$\sim(\sim p) \Leftrightarrow p$$

$$p \wedge \sim p \Leftrightarrow 0$$

$$p \vee \sim p \iff 1$$

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Section 2.5: cont'd

examples: $q \wedge \sim q \iff 0$

$$ABC + \overline{ABC} = 1$$

note: for MATH 156, you may <u>omit</u> explicitly writing the commutative and associative laws as separate step

example: simplify $0 \vee p$

| | | |
|---|---|---|
| very careful solution (has extra steps that you do not need) | $p \vee 0$ | commutative |
| | $p$ | identity |

| | | |
|---|---|---|
| totally acceptable Math 156 solutions | $p$ | identity |

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example: simplify using the LOL

$$(\sim p \vee 0) \wedge (q \vee \sim q) \wedge (1 \vee r)$$

| | |
|---|---|
| $\sim p \wedge (q \vee \sim q) \wedge 1$ | identity |
| $\sim p \wedge 1 \wedge 1$ | complement |

$\sim p \quad \wedge \quad 1 \quad \wedge \quad 1$ complement

$(\sim p \quad \wedge \quad 1) \quad \wedge \quad 1$ associative (can skip)

$\sim p \quad \wedge \quad 1$ identity

$\sim p$ "

example: $(p \wedge \sim p) \quad \vee \quad (p \vee \sim p)$

$0 \quad \vee \quad 1$ complement

$1$ identity, complement, definition of "or"

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simplify (brainteaser) $\sim(p \vee (q \wedge \sim r)) \quad \wedge \quad (p \vee (q \wedge \sim r))$

$0$

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simplify $A(\bar{B} B) \quad + \quad B(A + \bar{A})$

$A \cdot 0 \quad + \quad B \cdot 1$ complement

$0 \quad + \quad B$ identity

$B$ "

example: write a simplified expression for each of the following and identify which law you've used

1. $\sim r \wedge \sim r \iff \sim r$ idempotent
2. $ABC + ABC = ABC$ "
3. $\overline{ABC} + ABC = 1$ complement
4. $0 + \overline{B} = \overline{B}$ identity

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summary:

| | | |
|---|---|---|
| | identity laws | involve ones and zeros |
| | idempotent | involve a variable and/or itself |
| | complement | involves negation |

commutative
associative } can omit