

Section 2.6: More LOL

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De Morgan's:

$$\overline{AB} = \bar{A} + \bar{B}$$

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

examples: use De Morgan's to rewrite:

① $\overline{B+C} = \bar{B}\bar{C}$

② $\overline{\bar{B}+C} = B\bar{C}$

③ $\overline{\bar{B}+\bar{C}} = BC$

④ $\overline{\text{☺} + \text{☹}} = \overline{\text{☺}} \overline{\text{☹}}$

⑤ $\overline{A\bar{C}} = \bar{A}+C$

⑥ $\bar{A}C = \overline{A+\bar{C}}$

when would you see this in code

if $(x=5 \text{ or } y=2)$ then do \textcircled{A}
else do \textcircled{B} ↗

under what conditions does the

"else clause" with action B occur?

\Rightarrow When $(x = 5 \text{ or } y = 2)$ is FALSE
in other words, when $x \neq 5$ AND $y \neq 2$

Distributive:

$$A(B + C) = AB + AC$$

$$A + BC = (A + B)(A + C)$$

example: rewrite the following using the distributive law

$$\textcircled{1} \quad \bar{C}(A + C) = \bar{C}A + \bar{C}C$$

$$\textcircled{2} \quad (A + B)(A + \bar{B}) = A + B\bar{B}$$

$$\textcircled{3} \quad \bar{B} + \bar{A}\bar{C} = (\bar{B} + \bar{A})(\bar{B} + \bar{C})$$

$$\textcircled{4} \quad \overline{AB}(B + \bar{C}) = \overline{AB}B + \overline{AB}\bar{C}$$

Absorption:

$$A(A + B) = A$$

$$A(\bar{A} + B) = AB$$

$$A + AB = A$$

$$A + \bar{A}B = A + B$$

example: use the absorption laws to rewrite the following:

$$\textcircled{1} \quad \bar{C} (\bar{C} + A) = \bar{C}$$

$$\textcircled{2} \quad \bar{C} (C + A) = \bar{C} A$$

$$\textcircled{3} \quad AB + ABC = AB$$

$$\textcircled{4} \quad \bar{B} + BA = \bar{B} + A$$

$$\textcircled{5} \quad \overline{AB} + ABC = \overline{AB} + C$$

simplify

$$(p \vee q) \wedge (p \vee r) \Leftrightarrow p \vee (q \wedge r)$$

$$(\sim p \vee \sim q) \wedge (p \vee \sim q)$$

$$\sim q \vee (\sim p \wedge p)$$

distributive

$$\sim q \vee 0$$

complement

$$\sim q$$

identity

simplify

$$AB (\bar{A} + \bar{B})$$

method #1:

$$AB\bar{A} + AB\bar{B}$$

distributive

$$B \cdot 0 + A \cdot 0$$

complement

$$0 + 0$$

identity

$$0$$

one of $\begin{cases} \text{idempotent} \\ \text{identity} \\ \text{definition of "or"} \end{cases}$

method #2

$$AB(\bar{A} + \bar{B})$$

$$AB \overline{AB}$$

De Morgan's

$$0$$

complement

method #3

$$AB(\bar{A} + \bar{B})$$

$$A B \bar{A}$$

absorption

$$A(\bar{A} + B) = AB$$

$$B \cdot 0$$

complement

$$0$$

identity

prove:

$$\overline{B \cdot 0} = \overline{\bar{A}} + \overline{\bar{A} \bar{B}}$$

$$\bar{0} = \bar{\bar{A}} + \overline{\bar{A} \bar{B}}$$

identity

$$1 = \bar{\bar{A}} + \overline{\bar{A} \bar{B}}$$

definition of "not"

$$1 = \underbrace{\bar{\bar{A}} + A}_{1} + \bar{B}$$

DeMorgan's

$$1 = 1 + \bar{B}$$

complement

$$1 = 1 \quad \text{identity}$$

QED

simplify:

$$\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$$

method #1:

$$\bar{B}\bar{A} + \bar{A} \quad \text{absorption}$$

$$\bar{A} \quad \text{"}$$

method #2

$$\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$$

$$\bar{B}\bar{A} + \bar{B}B + \bar{A}\bar{A} + \bar{A}B \quad \text{distributive}$$

$$\bar{B}\bar{A} + 0 + \bar{A}\bar{A} + \bar{A}B \quad \text{complement}$$

$$\bar{B}\bar{A} + \bar{A}\bar{A} + \bar{A}B \quad \text{identity}$$

$$\bar{B}\bar{A} + \bar{A} + \bar{A}B \quad \text{idempotent}$$

$$\bar{A} + \bar{A}(\bar{B} + B) \quad \left\{ \begin{array}{l} \leftarrow \text{distrib} \\ \rightarrow \bar{B}\bar{A} + \bar{A} \quad \text{absorption} \end{array} \right.$$

$$\bar{A} + \bar{A} \cdot 1 \quad \left\{ \begin{array}{l} \text{complement} \\ \bar{A} \quad \text{absorption} \end{array} \right.$$

$$\bar{A} + \bar{A} \quad \text{identity}$$

$$\bar{A} \quad \text{idempotent}$$

method #3:

$$\bar{B} (\bar{A} + B) + \bar{A} (\bar{A} + B)$$

$$(\bar{A} + B)(\bar{B} + \bar{A})$$

distributive

$$\bar{A} + B\bar{B}$$

"

$$\bar{A} + 0$$

complement

$$\bar{A}$$

identity