

Section 2.6: More LOL

Wednesday, January 31, 2024 11:30 AM

De Morgan's:

$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{A+B} = \overline{A} \overline{B}$$

examples: use DeMorgan's to rewrite:

$$(1) \quad \overline{B+C} = \overline{B} \overline{C}$$

$$(2) \quad \overline{\overline{B}+C} = B \overline{C}$$

$$(3) \quad \overline{\overline{B} + \overline{C}} = B C$$

$$(4) \quad \overline{\text{😊} + \text{😞}} = \overline{\text{😊}} \overline{\text{😞}}$$

$$(5) \quad \overline{A \overline{C}} = \overline{A} + C$$

$$(6) \quad \overline{\overline{A} C} = \overline{A + \overline{C}}$$

when would you see this in code? (will not be tested)

if $x > 2$ or $y < 3$ then print "Hello world"
else print "Goodnight Moon"

under what conditions does "Goodnight Moon"
get printed?

when $(x > 2 \text{ or } y < 3)$ is FALSE

in other words, when $x \leq 2$ AND $y \geq 3$

Distributive

$$A(B + C) = AB + AC$$

$$A + BC = (A + B)(A + C)$$

examples: rewrite the following using the distributive law:

$$(1) \quad \bar{C}(A + C) = \bar{C}A + \bar{C}C$$

$$(2) \quad (A + B)(A + \bar{B}) = A + B\bar{B}$$

$$(3) \quad \bar{B} + \bar{A}\bar{C} = (\bar{B} + \bar{A})(\bar{B} + \bar{C})$$

$$(4) \quad \overline{AB}(B + \bar{C}) = \overline{AB}B + \overline{AB}\bar{C}$$

$$(5) \quad AB + B\bar{C} = B(A + \bar{C})$$

Absorption:

$$A(A + B) = A$$

$$A(\bar{A} + B) = AB$$

$$A + AB = A$$

$$A + \bar{A}B = A + B$$

example: use the absorption laws to rewrite the following:

$$(1) \quad \bar{B} + \bar{B}A = \bar{B}$$

$$(2) \quad \bar{C}(\bar{C} + \bar{A}) = \bar{C}$$

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$$(2) \quad C(C + \bar{A}) = C$$

$$(3) \quad \bar{C}(C + \bar{A}) = \bar{C}\bar{A}$$

Section 2.6: cont'd:

$$(4) \quad AB + ABC = AB$$

$$(5) \quad \bar{B} + BA = \bar{B} + A$$

$$(6) \quad \bar{A}B + ABC = \bar{A}B + C$$

simplify

$$(p \vee q) \wedge (p \vee r) \Leftrightarrow p \vee (q \wedge r)$$

$$(\sim p \vee \sim q) \wedge (p \vee \sim q)$$

$$\sim q \vee (\sim p \wedge p)$$

$$\sim q \vee 0$$

$$\sim q$$

distrib

complement

identity

simplify:

$$AB(\bar{A} + \bar{B})$$

method #1:

$$AB\bar{A} + AB\bar{B}$$

$$B \cdot 0 + A \cdot 0$$

$$0 + 0$$

distributive

complement

identity

0

one of { identity, idempotent, def of "or" }

$$AB(\bar{A} + \bar{B})$$

method #2:

$$AB \overline{AB}$$

De Morgan's

0

complement

$$AB(\bar{A} + \bar{B})$$

$$A(\bar{A} + B) = AB$$

method #3:

$$A B \bar{A}$$

absorption

$$B \cdot 0$$

complement

0

identity

prove:

$$\overline{B \cdot 0} = \bar{A} + \overline{A \bar{B}}$$

$$\bar{0} = \bar{A} + \overline{A \bar{B}}$$

identity

$$1 = \bar{A} + \overline{A \bar{B}}$$

definition of "not"

$$1 = \bar{A} + A + B$$

De Morgan's

$$1 = 1 + B$$

complement

$$1 = 1$$

identity

QED

simplify $\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$

method #1

$$\bar{B}\bar{A} + \bar{A}$$
$$\bar{A}$$

absorption

"

$$\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$$

method #2:

$$\bar{B}\bar{A} + \bar{B}B + \bar{A}\bar{A} + \bar{A}B$$

distributive

$$\bar{B}\bar{A} + 0 + \bar{A}\bar{A} + \bar{A}B$$

complement

$$\bar{B}\bar{A} + \bar{A}\bar{A} + \bar{A}B$$

identity

$$\bar{B}\bar{A} + \bar{A} + \bar{A}B$$

idempotent



$$\bar{A} + \bar{A}(\bar{B} + B)$$

distrib

$$\bar{A} + \bar{A} \cdot 1$$

complement

$$\bar{A} + \bar{A}$$

identity

$$\bar{A}$$

idempotent

$$\bar{B}\bar{A} + \bar{A}$$

absorption

$$\bar{A}$$

"

$$\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$$

method #3

$$(\bar{A} + B)(\bar{B} + \bar{A})$$

distrib

method #3

$$(\bar{A} + B)(\bar{B} + \bar{A})$$

distrib

$$\bar{A} + B\bar{B}$$

"

$$\bar{A} + 0$$

complement

$$\bar{A}$$

identity