Section 2.7: The Conditional

Tuesday, February 06, 2024 10:45 AM

conditional:

$$p \rightarrow q$$

"p implies q"

"if p, then q"

example: "If it is January, then it is winter."

---

example: If Barney is a dog, then Barney has four legs. ← true

Answer the following questions with "yes", "no", or "maybe."

a) Barney is a dog. Does he have four legs? Y

b) Barney is not a dog. Does he have four legs? M

c) Barney has four legs. Is he a dog? M

d) Barney does not have four legs. Is he a dog? N

---

true: "If Snarks are Boojums (p), then the Bellman is incorrect (q)."

- we cannot have p true and q false

Which of the following cannot occur?

a) Snarks are Boojums (p true) and the Bellman is incorrect.

p true

a) Snarks are Boojums and the Bellman is incorrect.

b) Snarks are not Boojums and the Bellman is incorrect.

q false

c) Snarks are not Boojums and the Bellman is correct.

p true q false

(d) Snarks are Boojums and the Bellman is correct.

Section 2.7: 2024/02/07

p q

example: If Pat sleeps in, then she will be late for class. ⇐ true

for this conditional $p \to q$, write the converse $q \to p$. Is the converse logically equivalent to the conditional?

converse: $q \to p$

If Pat is late for class, then she slept in.

logically equivalent? no!

so, how can we prove it? truth table

p	q	p→q	q→p
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

not same

not same

$p \to q$ is not logically equivalent to $q \to p$

$$(p \to q) \not\Longleftrightarrow (q \to p)$$

example: consider the conditional $p \to q$. Is it logically equivalent to the contrapositive $\sim q \to \sim p$? Justify your answer using a truth table.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

Yes

or, if you insist $(p \to q) \Longleftrightarrow (\sim q \to \sim p)$

---

example: write the contrapositive $\sim q \to \sim p$ for the following conditional $p \to q$.

"If I live in Saanich or Esquimalt, then I live in BC."

answer: If I don't live in BC, then I don't live in Saanich AND I don't live in Esquimalt.

stealth DeMorgan's!

why?

$\overline{\text{Saanich or Esquimalt}} = \overline{\text{Saanich}}$ and $\overline{\text{Esquimalt}}$

another perfectly acceptable answer would be:

If I don't live in BC, then I live in neither Saanich nor Esquimalt.

why? NOR = "not or"

---

what about the inverse, $\sim p \to \sim q$?

conditional	$p \to q$	
converse	$q \to p$	equivalent
contrapositive	$\sim q \to \sim p$	equivalent
inverse	$\sim p \to \sim q$	

---

the "or" form of the conditional:

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

$$(p \to q) \iff (\sim p \vee q)$$

example: for the conditional $p \to q$, rewrite it in the "or" form, $\sim p \vee q$.

If Pat sleeps in, then she will be late for class.

answer: Pat didn't sleep in or she was late for class.

---

digression: why do we care?

pseudocode:

```
if x > 3 then y = 4
print y
```

question: if the output is "4", was $x > 3$?

consider:

```
x = 5
y = 7

if x > 3 then y = 4
print y
```

output 4

```
x = 2
y = 4

if x > 3 then y = 4
print y
```

output 4

answer to our question
is MAYBE