Section 2.8: The Biconditional

Wednesday, February 07, 2024 11:38 AM

biconditional:

If you get 90% or higher in Math 156, then you will get an A+ and vice versa.

If and only if you get 90% or higher in Math 156, then you will get an A+

what does it mean?

"if p, then q and if q, then p"

or "if p, then q and if $\neg p$, then $\neg q$"

or "p and q are both true or they are both false"

notation: $p \leftrightarrow q$

"if and only if p, then q" (iff p then q)

"if p then q and vice versa"

---

example: Consider the following conditional statements. Do they still make sense when written as a biconditional?

(1) If you eat at Joe's, then you will have a good meal.

No

a good meal.

No

(2) If two lines are perpendicular, then they meet at a right angle.

Yes

(3) If today is Wednesday, then tomorrow is Thursday.

Yes

(4) If the battery is drained, then the car will not start.

No

---

truth table:

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

example: The following statement is true:

If and only if tulips are yellow, then goats are happy.

a) Tulips are yellow. Are goats happy? Yes

b) Tulips are not yellow. Are goats happy? No

c) Goats are unhappy. Are tulips yellow? No

d) Goats are happy. Are tulips yellow? Yes

rule: p and q are both true or both false

note: there are no MAYBEs here!

example: Is the biconditional $p \Leftrightarrow q$ logically equivalent to

$(p \to q) \wedge (q \to p)$?

Use a truth table to justify your answer.

p	q	$p \Leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

Yes

if you insist, $(p \Leftrightarrow q) \Leftrightarrow ((p \to q) \wedge (q \to p))$