Section 3.2: Arithmetic Sequences and Series

Tuesday, February 13, 2024 10:59 AM

examples:

	pattern?
① $2, 5, 8, \ldots$	add 3
② $0.4, 0.5, 0.6, \ldots$	add 0.1
③ $5, -5, -15, \ldots, -205$	add -10
④ $-1, -\frac{3}{2}, -2, -\frac{5}{2}, \ldots, -10$	add $-\frac{1}{2}$

arithmetic sequence ≡ a sequence in which you find the next term by adding a constant to the previous term

common difference
d

why? to find d, subtract any term from the next term

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recursive form for arithmetic

example: write the recursive formula for the sequence

$0.4, 0.5, 0.6, \ldots$

answer:
$$\begin{cases} a_0 = 0.4 \\ a_n = a_{n-1} + 0.1 \quad \text{for } n \geq 1 \end{cases}$$

↑
or $n > 0$
or $n = 1, 2, 3, \ldots$

in general:

$\{$ a = ⟨insert first term here⟩

in general:

$$\begin{cases} a_m = \langle \text{insert first term here} \rangle & \\ a_n = a_{n-1} + d & \text{for } n \geq m+1 \\ & \text{or } n > m \end{cases}$$

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general formula: $2, 5, 8, \ldots$

①	②	③	④		ⓝ	
$2,$	$5,$	$8,$	$11,$	$\ldots,$	$a_n,$	$\ldots$
$2,$	$2+3,$	$2+2\cdot3,$	$2+3\cdot3,$		$2+(n-1)\cdot3$	

so for this particular example,

$$a_n = 2 + (n-1)\cdot 3$$

and in general $\boxed{a_n = a_m + (n-m)\,d \quad \text{for } n \geq m}$

note: to find the simplified general formula for $2, 5, 8, \ldots$

we should simplify
$$\begin{aligned} a_n &= 2 + (n-1)\,3 \\ &= 2 + 3n - 3 \\ &= 3n - 1 \end{aligned}$$

$$\boxed{a_n = 3n - 1 \quad \text{for } n \geq 1}$$

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example: find a general formula for the sequence

$$5, \ -5, \ -15, \ \ldots$$

simplify your answer and specify what values of the index to use

answer: this sequence is arithmetic

let's use m=0 as our starting index

$$a_0 = 5$$
$$d = -10$$

$$a_n = a_m + (n-m)d$$
$$a_n = 5 + (n-0)(-10)$$

$$\boxed{a_n = 5 - 10n \quad \text{for} \quad n \geq 0}$$

the advantage of the general formula is that calculating individual terms is more straightforward than for the recursive form

- to find 10000th in a recursive formula, you need to know the 9999th term, which requires the 9998th term, and so on

- to find the 10000th term in a general formula, just plug in for n

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example: for the arithmetic sequence in which the first term is 2 and the fiftieth term is 394, what is the common difference?

answer: $a_n = a_m + (n-m)d$

let $m=1$ $\quad a_1 = 2$

$a_{50} = 394$

$$394 = 2 + (50-1)d$$
$$394 = 2 + 49d$$
$$392 = 49d$$
$$d = 8$$

example: An arithmetic sequence has the seventy-fifth term equal to −437. If the common difference is −6, calculate the first term.

answer:

$m=0$

$$a_n = a_m + (n-m)d$$
$$-437 = a_0 + (74-0)(-6)$$

etc

$m=1$

$$a_n = a_m + (n-m)d$$
$$-437 = a_1 + (75-1)(-6)$$
$$-437 = a_1 - 444$$
$$a_1 = 7$$

example: Consider the sequence

$$18, 30, 42, \ldots, 582$$

How many terms?

$$a_n = a_m + (n-m)d$$
$$582 = 18 + (n-1)12$$
$$564 = 12(n-1)$$
$$47 = n-1$$
$$48 = n$$

$$\boxed{n = 48}$$

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arithmetic series:

$$2 + 5 + 8 + \ldots$$

for this series, calculate S_8

answer: $$S_8 = \underbrace{2 + \overbrace{5 + \underbrace{8 + \overbrace{11 + 14} + 17} + 20}^{25} + 23}_{25}$$

$$S_8 = 4 \times 25$$

number of pairs

sum of each pair

$$S_k = \frac{k}{2}(a_m + a_n)$$

$$k = n - m + 1$$

but what if k is odd?

half of 22

$$S_7 = \underbrace{2 + \overbrace{5 + \underbrace{8 + \textcircled{11} + 14}_{22} + 17}^{22} + 20}_{22}$$

so we have 3½ pairs whose sum is 22

→ still works!

summary:

$$\boxed{\begin{aligned} S_k &= \frac{k}{2}(a_m + a_n) \\ S_k &= \frac{k}{2}(2a_m + (n-m)d) \end{aligned}}$$

$k = n - m + 1$

$a_n = a_m + (n-m)d$

example: find the sum of the first 50 terms of

$$2 + 5 + 8 + \ldots$$

answer:

method #1: arithmetic with $d = 3$

$m = 1$
$n = 50$
$k = 50$

$$S_k = \frac{k}{2}(2a_m + (n-m)d)$$

$$S_{50} = \frac{50}{2}(2 \cdot 2 + (50-1)3)$$

$$= 3775$$

method #2: arithmetic with $d = 3$

$$S_k = \frac{k}{2}(a_m + a_n)$$

$$= \frac{50}{2}(2 + \quad)$$

↑

but what's this?

$$\begin{aligned} a_n &= a_m + (n-m)d \\ &= 2 + (50-1)3 \\ &= 149 \end{aligned}$$

$$\begin{aligned} S_{50} &= \frac{50}{2}(2+149) \\ &= 3775 \end{aligned}$$

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evaluate:

$$\sum_{j=4}^{50}(6j-3) = \overset{\textcircled{4}}{(6\cdot4-3)} + \overset{\textcircled{5}}{(6\cdot5-3)} + \overset{\textcircled{6}}{(6\cdot6-3)} + \dots + \overset{\textcircled{50}}{(6\cdot50-3)}$$

$$= 21 + 27 + 33 + \dots + 297$$

arithmetic with $d = 6$

$$\begin{aligned} S_k &= \frac{k}{2}(a_m + a_n) \\ &= \frac{47}{2}(21 + 297) \\ &= 7473 \end{aligned}$$

$$\begin{aligned} k &= n - m + 1 \\ &= 50 - 4 + 1 = 47 \end{aligned}$$

summary:

arithmetic:

$$\boxed{\begin{aligned} a_n &= a_m + (n-m)d \\ S_k &= \frac{k}{2}(a_m + a_n) \\ S_k &= \frac{k}{2}(2a_m + (n-m)d \end{aligned}}$$

for $n \geq m$

$\left.\right\}$ $k = n - m + 1$