Section 4.1: Big O and Rates of Growth

Wednesday, February 28, 2024 10:48 AM

example: suppose you want to own digital copies of a number of Star Trek episodes. You can either download them at one episode per hour or you can get overnight delivery from an online retailer (call it 24 hour delivery).

which method should you choose?

answer: it depends on how many episodes you want

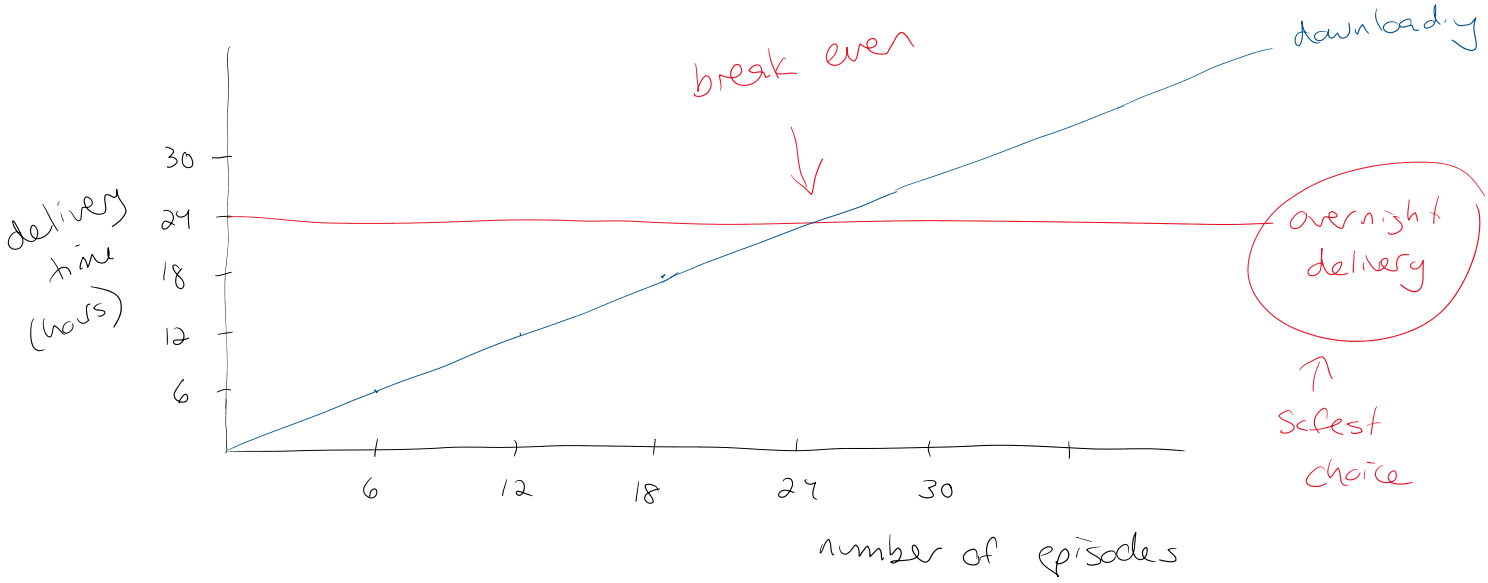
scenarios: we want minimum delivery time

few episodes (< 24), downloading is faster

break even ($= 24$), methods take same time

many episodes (> 24), overnight faster

and if you don't know how many episodes are needed, then the "many episodes" scenario is the safest approach



number of steps required for algorithms:

consider adding two 5-digit numbers:

```
  12345
+ 12345
```

best-case scenario:
5 additions (one for each column)

worst case:
5 additions
plus some "carrys"

consider multiplying two 5-digit numbers:

```
  12345
x 12345
 xxxxx
 xxxxx
 xxxxx
 xxxxx
 xxxxx
```

best case scenario:
25 multiplications
plus 9 column additions

worst case:
25 multiplications
plus 10 column additions
plus lots of "carrys"

the worrying part is the fact that the number of multiplications is the square of the number of digits

100 digits → 10 000 multiplications!
ouch!

addition: if you double the number of digits, you approximately double the number of operations

Big O notation: $O(n)$

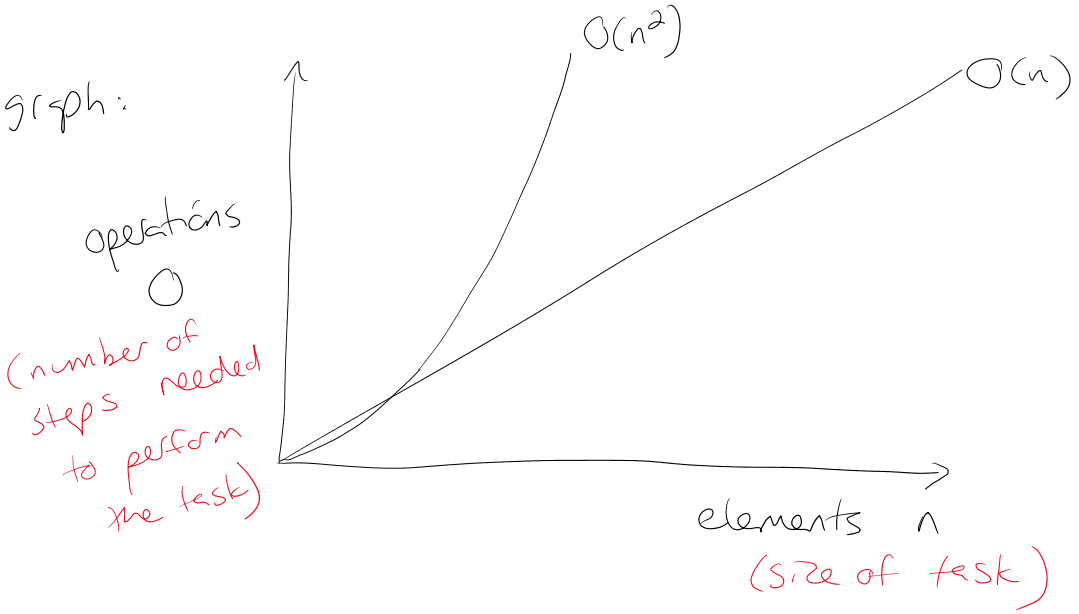
"of order n"

multiplication: if you double the numbe of digits, you approximately quadruple ($\times 4$) the numbe of operations

$O(n^2)$

note: in Math 156, I will not ask you to analyse an algorithm to figure out Big O

graph:



most efficient method for large n:
- this is the method with the

smallest number of steps when n is large

- to find it, go to the right side of the graph. Start at x-axis and go straight up. First line you reach is the most efficient method.

note that even if the straight line had a very steep slope, the $O(n^2)$ graph will cross it for sufficiently large n

Big O is only concerned with the number of operations for very large n

"worst case scenario"