# Section 4.2: Factorial and Exponential Growth

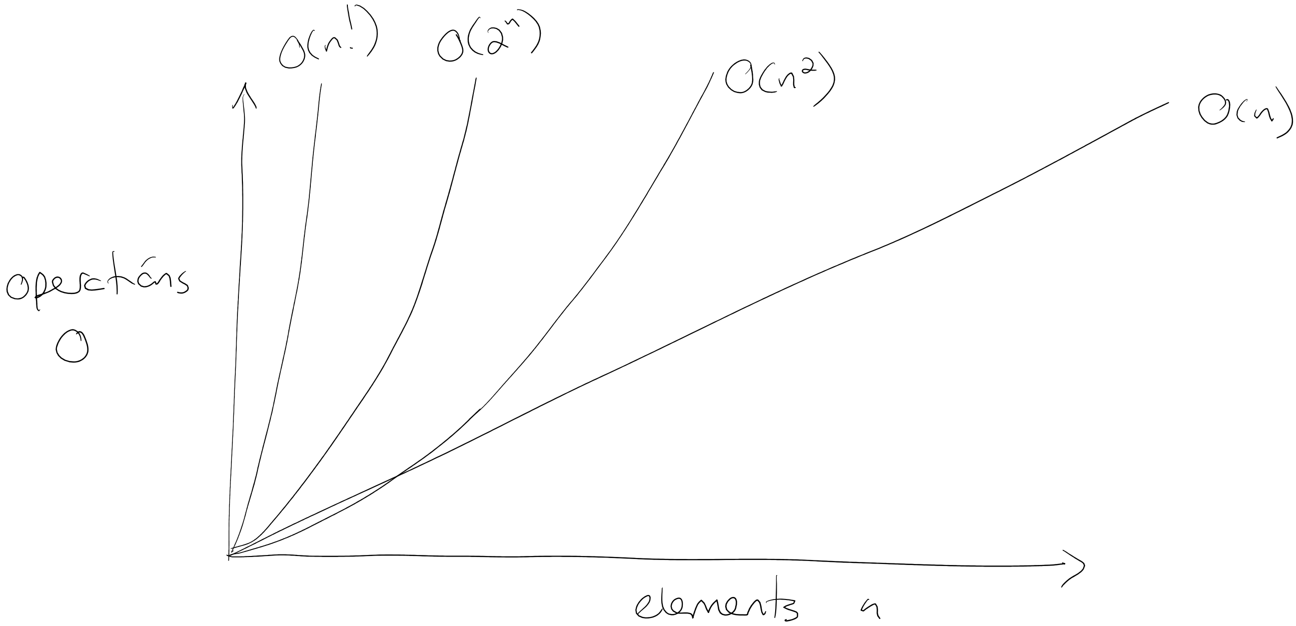
Wednesday, February 28, 2024 11:57 AM

factorial: $3! = 3 \cdot 2 \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$n! = n(n-1)(n-2) \ldots \cdot 3 \cdot 2 \cdot 1$

| $n$ | polynomial $n^2$ | exponential $2^n$ | factorial $n!$ |
|---|---|---|---|
| 1 | 1 | 2 | 1 |
| 2 | 4 | 4 | 2 |
| 3 | 9 | 8 | 6 |
| 4 | 16 | 16 | 24 |
| 5 | 25 | 32 | 120 |
| 10 | 100 | 1024 | 3 628 800 |
| 100 | 10 000 | $1.267 \times 10^{30}$ | $9.33 \times 10^{157}$ |




what I will be testing you on is ranking the various orders

$O(n)$, $O(n^2)$, etc

various orders

$O(n)$, $O(n^2)$, etc

so that you know the shape of each graph and are able to tell which one is more efficient as $n$ gets large

## Big O for sums of different functions

what if your procedure requires $n^2+2n+5$ steps for a task of size $n$? what is Big O?

| $n$ | $n^2$ | $2n$ | $5$ | $n^2+2n+5$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 5 | 8 |
| 10 | 100 | 20 | 5 | 125 |
| 100 | 10 000 | 200 | 5 | 10 205 |
| 1000 | 1 000 000 | 2000 | 5 | 1 002 005 |

as $n$ gets large, the contributions to the total from $2n$ and $5$ become very small in comparison to the contribution from the $n^2$ term

for large $n$, $O(n^2+2n+5) \approx O(n^2)$

approximately equal to

to find Big O for the sum of different functions to find Big O for the sum of different functions

- locate in the sum the term that's growing the fastest
- remove any coefficients
- what's left is Big O

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example: consider procedures where the number of operations required for a task of size n is given below. Find Big O for each procedure.

a) $9n + 5$ answer: $O(n)$

b) $2^n + n^2$ $O(2^n)$

c) $4^2 + 4!$ these are just constants so $O(1)$

d) $n! + 4!$ $O(n!)$

e) $3n(n+1) = 3n^2 + 3n$ $O(n^2)$