

Section 6.4: Measures of Relative Standing

Wednesday, March 20, 2024 10:15 AM

measures of relative standing

- give an indication of the position of an individual data point with respect to the rest of the data

example: you ran a race and your time was 56 minutes

- is this good or bad?

it would be nice to have a single number that tells you how you did with respect to everyone else

measures: percentile (we will not cover)

Z-score

Z-score:

for populations, it's

$$Z = \frac{x - \mu}{\sigma}$$

data point you are

population mean

population std dev

point
you are
interested
in

11

for samples, it's

$$z = \frac{x - \bar{x}}{s}$$

← sample mean

↑ sample std dev

z is then the number of standard deviations above the mean that the data point is

- if z is negative, then the data point is below the mean

so, how likely are various z -scores?

notation: $|z|$ is the absolute value of z
(how far from the origin z is)

$$|3| = 3$$

$$|-3| = 3$$

$$|0| = 0$$

$$|z| > 2$$

for any data set, will happen
< 25% of the time (Chebyshev)

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< 25% of the time (Tcheby)

for unimodal and symmetrical, will
happen ~ 5% of the time
(Empirical)

so we say $|z| > 2$ is somewhat unlikely

$$|z| > 3$$

- for any data set / distribution,
will happen < 1% of the time
(Tcheby)

- for unimodal and symmetrical,
will happen ~ 0.3% of the time
(Empirical)

so we say $|z| > 3$ is very unlikely

Summary: $|z| > 2$ somewhat unlikely

$|z| > 3$ very unlikely

$|z| \leq 2$ likely

example:

Ten DVDs were picked at random from
Pat's collection and the number of
Academy Awards (Oscars) won by each
movie was recorded.

results: 0, 0, 1, 2, 0, 0, 2, 0, (1)

The standard deviation for this data set is 3.37. Calculate the z-score for any outliers and state whether these data points are likely or unlikely.

answer: $\bar{x} = 1.7$
 $s = 3.37$

$$z = \frac{x - \bar{x}}{s}$$

$$= \frac{11 - 1.7}{3.37}$$

$$\approx 2.761158$$

$$\approx 2.76$$

(in this course, we generally round z to 2 decimal places)

unlikely



note: Lord of the Rings: Return of the King was not a typical movie