Chapter 8: Probability

Wednesday, March 20, 2024 11:18 AM

Section 8.1: Counting Techniques

example: How many 4-digit positive integers are evenly divisible by 5?

answer: 1000, 1005, 1010, ..., 9995

note: this is an arithmetic sequence with $d = 5$

\# choices for each digit: $\underset{\substack{\text{choose}\\\text{from}\\1\text{ to }9}}{\underline{9}}\ \underbrace{\underline{10}\ \underline{10}}_{\substack{\text{choose from}\\0\text{ to }9}}\ \underset{\substack{\text{choose}\\\text{from}\\0\text{ or }5}}{\underline{2}}$

now multiply these choices together to get

\# choices $= 9 \cdot 10 \cdot 10 \cdot 2 = 1800$

this method only works when you rule out possibilities in one or more slots

works for "divisible by 5 or 2 or 10"

doesn't work for "divisible by 3 or 7 or ..."

multiplication rule:

suppose we have an event which is made up suppose we have an event which is made up of n different independent steps

total number of outcomes (ways the event can happen) = ____ × ____ × ____ × ... × ____

number of ways the first step can happen

2nd step

final step

example: How many different BC licence plates for cars are there?

patterns: LLL NNN
NNN LLL
LLN NNL

L = letter
N = number

(assume that all letters and numbers are used and ignore reserved words and personalized plates)

answer: top pattern: $\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}$

$= 26^3 \cdot 10^3$

$= 17\ 576\ 000$

so total plates for all patterns = $3(17\ 576\ 000)$

$= 52\ 728\ 000$

example: In the mythical Canadian province of Gondor, licence plates follow the pattern letter - letter - letter number - number. Due to recent political events, the letter combination EYE is no longer allowed. How many legal licence plates are there in Gondor?

answer: number of legal plates = total number possible − number of illegal

total number possible = $\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 26^3 \cdot 10^2$

illegal plates: $\underset{\text{"E"}}{\underline{1}}\ \underset{\text{"Y"}}{\underline{1}}\ \underset{\text{"E"}}{\underline{1}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 100$

number of legal plates $= 26^3 10^2 - 1^3 \cdot 10^2$

$= 1\ 757\ 500$

note: the reason you cannot just say

$\underline{25}\ \underline{25}\ \underline{25}\ \underline{10}\ \underline{10}$

is that you can still start with an E

(ECR 58 is okay)

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tip: when finding the number of allowed outcomes sometimes it's easier to calculate the total number of outcomes and subtract from that the number of outcomes that are not allowed

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the addition rule:

example: how many positive integers from 1 to 20 inclusive are

a) evenly divisible by 2?
b) " " 3?
c) " " 2 or 3?

answer: brute force method

divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 — 10

3: 3, 6, 9, 12, 15, 18 — 6

2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 — 13

note $13 \neq \underbrace{10 + 6}_{\text{contains duplicates}}$

but: $13 = \underset{\text{first group}}{10} + \underset{\text{second group}}{6} - \underset{\text{overlap}}{3}$

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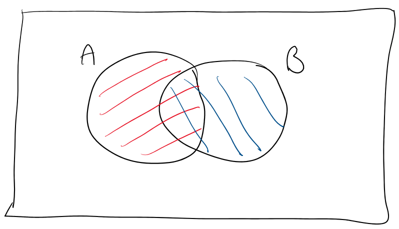
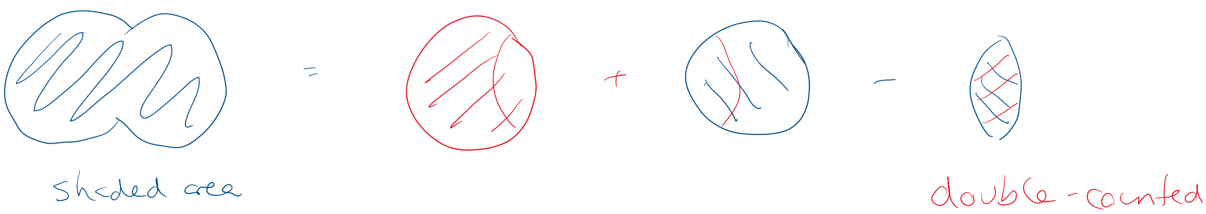
so $\quad n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

$\uparrow$ number of events containing A or B

$\uparrow$ can also be written as $n(AB)$

Venn diagram





example: How many 4-digit PINs

$\nwarrow$ personal identification number

a) start with 9 ?

b) end in 4 ?

c) start with 9 or end in 4 ?

d) start with 9 or 4 ?

answer:

a) start with 9 : $\quad \frac{1}{9}\ \underbrace{\underline{10}\ \underline{10}\ \underline{10}}_{\text{any digit}} = 10^3$ or 1000

b) end in 4 : same

c) start with 9 or end in 4

$n(\text{start } 9 \text{ or end } 4) = n(\text{start } 9) + n(\text{end } 4) - n(\text{both})$

$n(\text{both}) = \underset{9}{\underline{1}}\ \underset{\text{any}}{\underline{10}}\ \underset{\text{any}}{\underline{10}}\ \underset{4}{\underline{1}} = 100$ or 10^2

$n(\text{start } 9 \text{ or end } 4) = 1000 + 1000 - 100$

$= 1900$

d) start with 9 or 4 $= n(\text{start } 9) + n(\text{start } 4) - n(\text{both})$

$= 1000 + 1000 - \underline{\underline{0}}$

$= 2000$

or $\underset{9 \text{ or } 4}{\underline{2}}\ \underbrace{\underline{10}\ \underline{10}\ \underline{10}}_{\text{any}} = 2000$

example: How many 4-digit PINs are there if repetition of digits is not allowed?

once you've used a 3, you can't use it again

answer: $\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = 5040$

digression: will not be tested

there's! another! way! to! calculate! this!

$10 \cdot 9 \cdot 8 \cdot 7 = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{10!}{6!}$

number of choices = 10 (digits)
number of slots = 4

denominator is $(10-4)!$

this is called a permutation and on your calculator, it's the nPr button

so to get $_{10}P_4$ on your calculator

10 nPr 4 = 5040

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example: How many 5-character case-sensitive alphanumeric passwords are there

a) in total?
b) that contain at least one letter <u>and</u> at least one number?

answer: a) how many choices do we have have?

alphanumeric - both letters and numbers

case-sensitive - uppercase and lowercase letters are considered to be different (A versus a)

choices = 10 digits + 26 uppercase letters + 26 lowercase letters
= 62

5 characters password: $\underline{62}\ \underline{62}\ \underline{62}\ \underline{62}\ \underline{62} = 62^5$
or 916 132 832

b) want at least one letter <u>and</u> at least one number total allowed passwords = total possible − total not allowed

not allowed : all letters : 52^5
all numbers 10^5

total allowed passwords = $62^5 - 52^5 - 10^5$

= 535 828 800
or 5.36×10^8

one common mistake:

$\underline{52}\ \underline{10}\ \underline{62}\ \underline{62}\ \underline{62}$ ← this approach omits other valid passwords like AAAA2

↑
number is in last slot