Section 8.2: Classical Probability

Thursday, March 21, 2024 2:16 PM

classical probability: if all outcomes are equally likely, then the probability of event E happening is

$$P(E) = \frac{n(E)}{n_{tot}}$$

"n of E", number of ways E can happen

n_{tot}: total number of outcomes

$P(E)$: probability of event E happening

"P of E"

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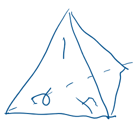
example: If you roll two fair 4-sided dice, what's the probability that the sum of the rolls is 3 or less?



fair = all rolls are equally likely
(not fair = weighted or loaded)

note: one die, two dice (dice = plural)

answer: sample space (set of all possible outcomes)

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

16 outcomes in total

$$P(\text{sum} \le 3) = \frac{n(\text{sum} \le 3)}{n_{tot}}$$

$$= \frac{3}{16}$$

what, then, is the probability that the sum will be greater than 3?

answer: $P(\text{sum} > 3) = 1 - P(\text{sum} \le 3)$

$$= 1 - \frac{3}{16}$$

$$= 13/16$$

why? $P(A) = 1 - P(\bar{A})$

what's the probability of rolling a sum of 5?

answer: $P(\text{sum} = 5) = \frac{n(\text{sum } 5)}{n_{tot}}$

$$= \frac{4}{16} = \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$$

what's the probability that at least one die shows the number 3?

$$P(\text{at least one } 3) = \frac{7}{16}$$

probability demonstration with two 4-sided dice:

what are the rolls (individual dice)?

what is the sum of the two dice?

what are the rolls of individual dice?

1	$\cancel{||||}$ |
2	$\cancel{||||}$ |
3	$\cancel{||||}$ ||
4	$\cancel{||||}$ ||||

what is the sum of the two dice?

2	|
3	
4	||||
5	||
6	|||
7	|
8	||

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for event A, the complement can be written as

$\bar{A}$

event in which A does not happen

other notations: A^c, A', $\sim A$, $\neg A$

the complement of A is the set of outcomes in which A does not occur

$P(A) + P(\bar{A}) = 100\%$

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some vocabulary:

experiment - process by which an observation (measurement) is obtained

example: you roll a six-sided die

simple event: the outcome observed on a single repetition of the experiment single repetition of the experiment

example: you roll a 2

compound event - a collection of simple events (sometimes just called an event)

example: you roll an even number

sample space - the complete list of all outcomes (list of all simple events)

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two properties of probability:

1. $0 \le P(A) \le 1$
2. the sum of the probabilities for all possible outcomes is 1

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if you are having trouble generating the sample space, another approach is a tree diagram

example: write out the sample space for flipping a coin three times and recording the result

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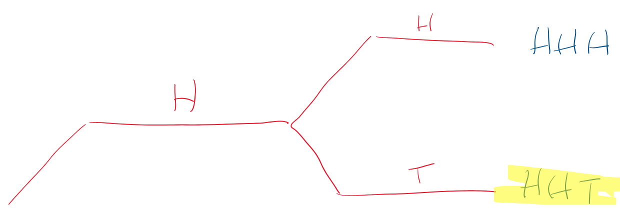
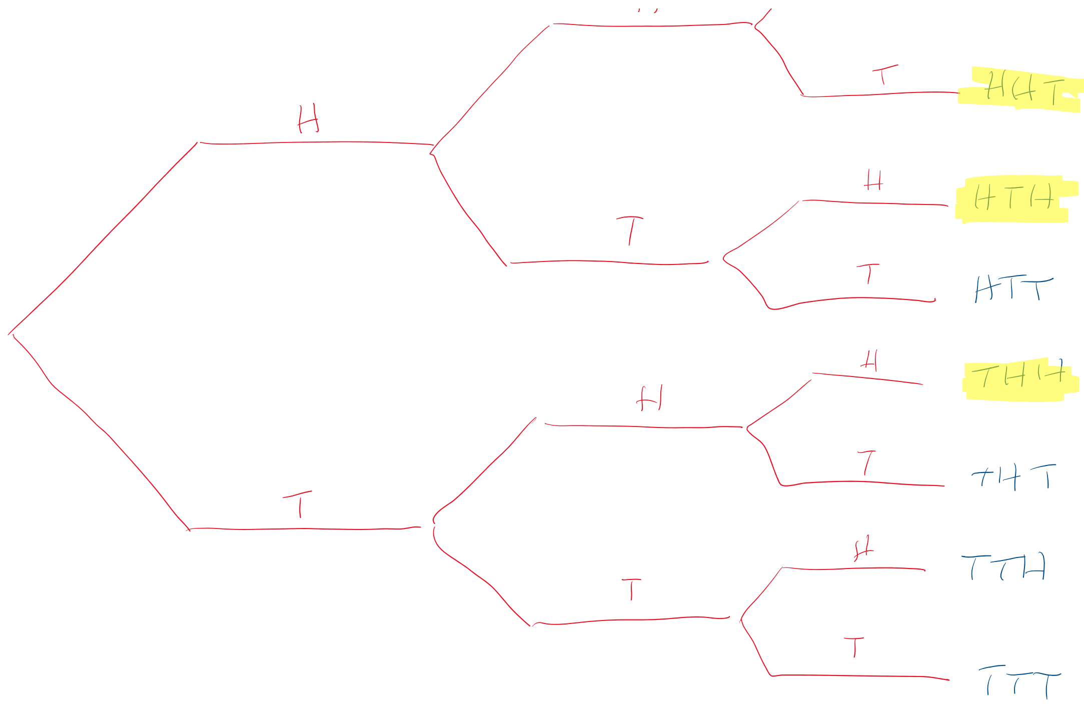
answer:



answer:



so then, how many different ways can you get only one tail?

3

if the coin is fair, then what is the probability of getting exactly one tail?

$\frac{3}{8}$

example: At the Red Barn Market, you can get an ice cream cone with two scoops of ice cream. Let's assume that you have to choose two different flavours for your scoops and it doesn't matter which flavour is on top. Let's further assume that when averaged over all customers, each flavour is equally likely.

flavours available: chocolate
vanilla

raspberry
strawberry

a) how many different ice cream cones are possible?

brute force :

CV VR RS
CR VS
CS

6 different cones

b) what's the probability that a random customer will order chocolate as one of the two scoops?

$$P(C) = \frac{n(C)}{n_{tot}} = \frac{3}{6} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

c) what's the probability that a random customer will order chocolate and vanilla?

$$P(CV) = \frac{n(CV)}{n_{tot}} = \frac{1}{6}$$

d) what's the probability that a random customer will order chocolate or vanilla?

$$P(C \text{ or } V) = \frac{n(C \text{ or } V)}{n_{tot}} = \frac{5}{6}$$

e) calculate (d) again using a different method

$$P(C \text{ or } V) = P(C) + P(V) - P(\text{both})$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$$

f) calculate (d) again using <u>yet another</u> method!

$$P(C \text{ or } V) = 1 - P(\overline{C \text{ or } V})$$

$$= 1 - P(RS)$$

$$= 1 - 1/6 = 5/6$$

summary of rules:

$$P(\text{event}) = \frac{n(\text{event})}{n_{\text{total}}}$$

$$P(\text{event}) = 1 - P(\overline{\text{event}})$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

this term can be zero
– then the events are said to be "mutually exclusive"

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contingency table

example: suppose we survey students in Math 156 to find out whether they like coffee and/or spicy food

		C like coffee	$\overline{C}$ don't like coffee	
S	like spicy food	13	5	18
$\overline{S}$	don't like spicy food	2	4	6

S	like spicy food	13	5	18
$\bar{S}$	don't like spicy food	2	4	6
		15	9	total 24

a) what's the probability that a randomly chosen student likes coffee and spicy food?

$$P(CS) = \frac{n(CS)}{n_{tot}} = \frac{13}{24}$$

b) what's the probability that a randomly chosen student likes coffee?

$$P(C) = \frac{n(C)}{n_{tot}} = \frac{15}{24} = \frac{5}{8}$$

c) what's the probability that a randomly chosen student doesn't like coffee?

$$P(\bar{C}) = 1 - P(C) = 1 - \frac{5}{8} = \frac{3}{8}$$

d) what's the probability that a randomly chosen student likes coffee or spicy food?

$$P(C \text{ or } S) = \frac{n(C \text{ or } S)}{n_{tot}}$$

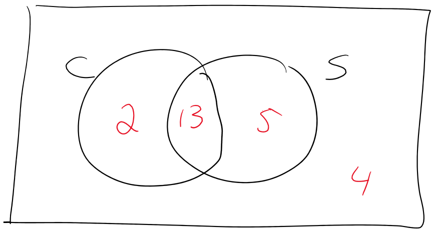
$$= \frac{13 + 5 + 2}{24}$$

$$= \frac{20}{24} = \frac{5}{6}$$

note:

	C	$\bar{C}$
S	13	5
$\bar{S}$	2	4

S	13	5
$\bar{S}$	2	4



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example: In a class of 45 students, 26 have jobs and 17 have cars. Of those who don't have a car, 10 have jobs.

a) Fill out a contingency table for this situation.

	jobs	no jobs	
car	16	1	17
no car	10	18	28
	26	19	total: 45

b) Find the probability that a student has a car or a job.

$$P(C \text{ or } J) = \frac{n(C \text{ or } J)}{n_{tot}} = \frac{16+10+1}{45}$$

$$= \frac{27}{45} = \frac{3}{5} \text{ or } 0.6 \text{ or } 60\%$$

c) Find the probability that a student has a car but not a job.

$$P(C\bar{J}) = \frac{n(C\bar{J})}{n_{tot}} = \frac{1}{45} \text{ or } 2.\bar{2}\%$$

(can also

n)

$$P(C\bar{J}) = \frac{n(C\bar{J})}{n_{tot}} = \frac{1}{45} \approx 0.02$$

(can also say just 2%)

note: we could also represent this contingency table using percentages:

	J	$\bar{J}$	
C	36%	2%	38%
$\bar{C}$	22%	40%	62%
	58%	42%	total: 100%