Section 9.1: Continuous Random Variables

Wednesday, March 27, 2024 11:34 AM

recall: discrete random variables

↳ can only take on certain values

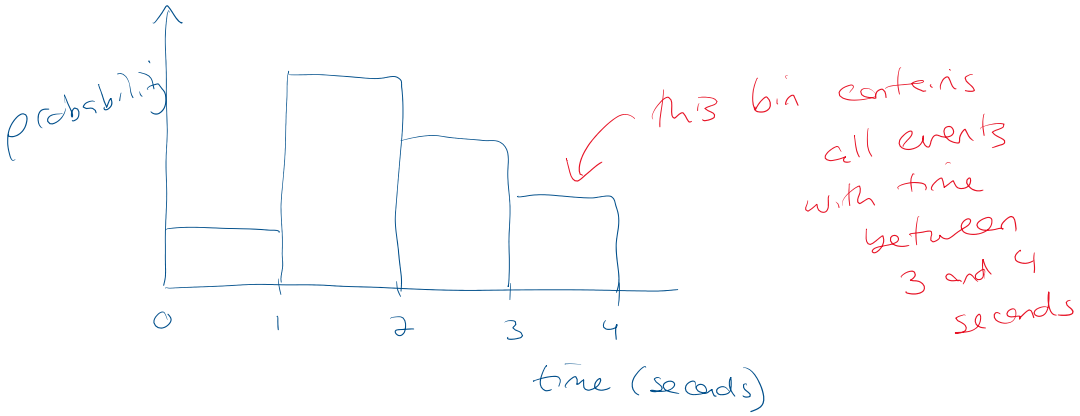
now: continuous random variables

↳ variable can take on an infinite number of values and can always split the difference between any two values

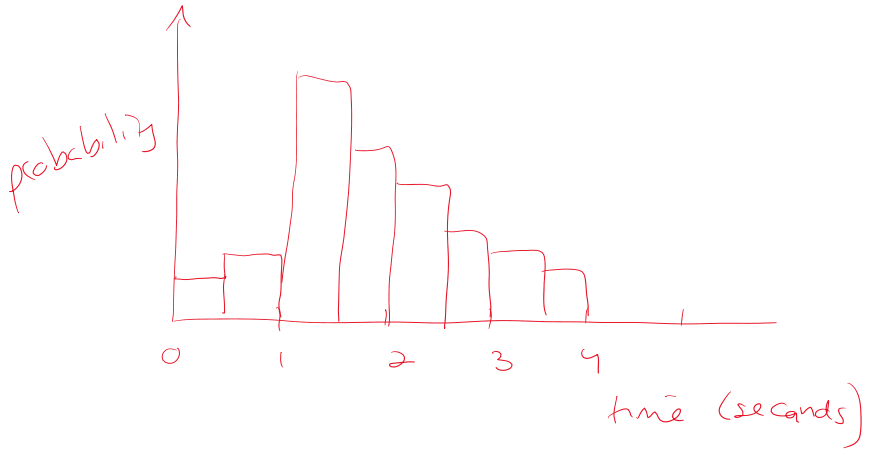
example: if your variable is time and you have measured the values 2.78 seconds and 2.79 seconds you could, in theory, also measure a value of 2.785 seconds

but what does a probability distribution look like if you have an infinite number of values that your variable could take?

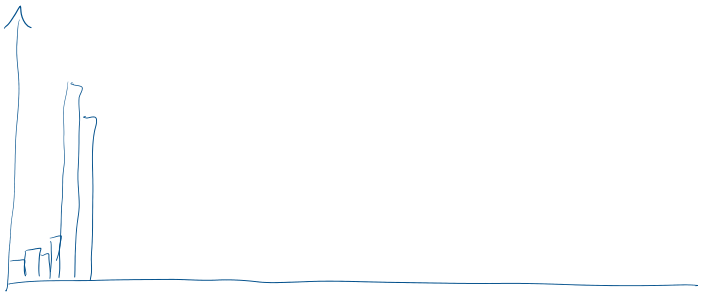
- you could group the data into "bins" and make a histogram



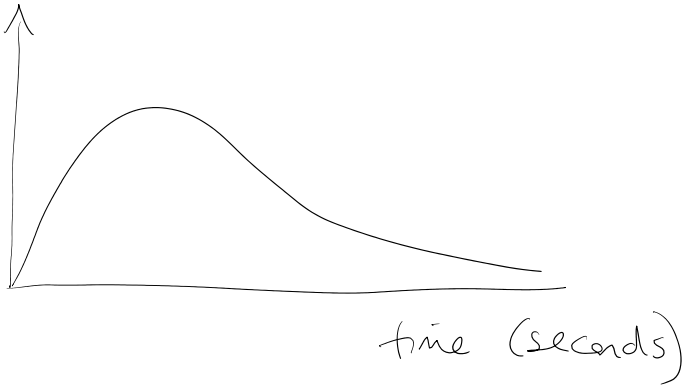
or



or



and eventually, the rectangles will get so small that you can't even see them and you'll get a smooth continuous curve



this smooth curve is called a density curve

properties of density curves:

(1) it is always on or above the x-axis (y-value is never negative)

(2) the y-axis isn't probability any more

⇒ rather, the scale on the y-axis is chosen such that the area underneath the curve is exactly equal to one (100%)

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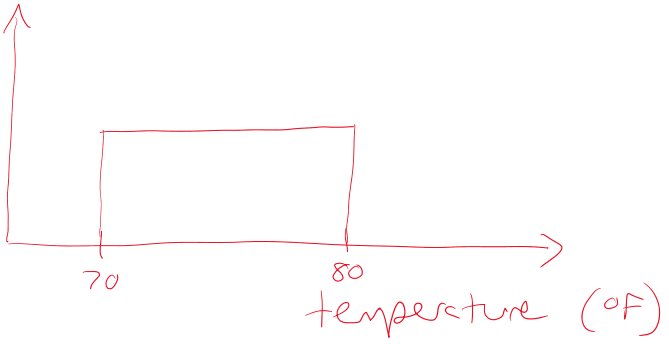
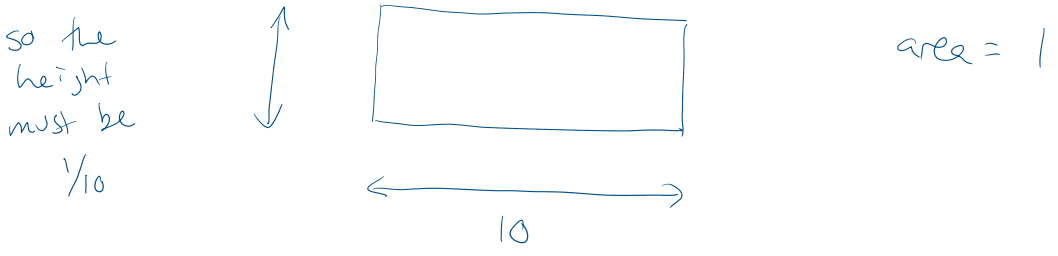
example: the continuous uniform probability distribution

In March in Pasadena, CA, the temperature during the day is always between 70°F and 80°F with an equal probability of any temperature within that range.

a) what does the density curve look like?



b) what is the height of the rectangle?



c) what is the average temperature in March in Pasadena?

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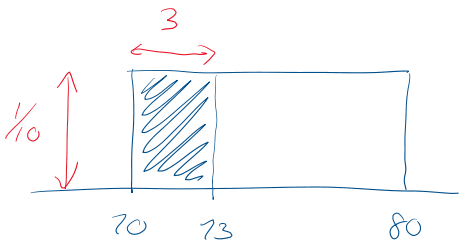
by symmetry, the mean temp is 75°F

d) What is the probability that the temperature is less than 73°F?



$$\text{area} = \frac{1}{10} \cdot 3 = \frac{3}{10} \quad \text{or } 30\%$$

e) What is the probability that the temperature is exactly equal to 73°F?

73.000000000000000000000000000...

The probability is exactly equal to zero

(when there are an infinite number of values you could measure, the chance of getting any one value is identically zero)