Section 1.4: Converting From Decimal

Friday, September 08, 2023 11:54 AM

we've converted from other bases to decimal - how do we go the other way?

background:

whole numbers 0, 1, 2, 3, ...

integers ... -3, -2, -1, 0, 1, 2, 3, ...

modular arithmetic

$\frac{7}{4} = 1\frac{3}{4}$ or 1.75

but if we stay with integers

$\frac{7}{4}$ has a quotient of 1 plus 3 left over

whole number or integer part

quotient Q remainder R

how do you find Q and R on a calculator

$\frac{7}{4} = 1.75$ on calculator

integer part is 1

non-integer part 0.75

remainder = 4 × 0.75 = 3

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note: how do you get the non-integer part quickly? subtract the integer part

example: find Q and R for

a) $50 \div 4 = 12.5$ — Q = 12, R = 4 × 0.5 = 2

b) $92 \div 8 = 11.5$ — Q = 11, R = 8 × 0.5 = 4

c) $133 \div 16 = 8.3125$ — Q = 8, R = 16 × 0.3125 = 5

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application: unit conversion

time = 45 days

= 6 weeks (quotient) and 3 days (remainder)

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digression: will not be tested in this class (Math 156)

in computing, the modulus (or modulo) function computes the remainder left after dividing one integer by another

mod(5, 3) = 2

5 mod 3 = 2

5 % 3 = 2 (Java)

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recall: converting from octal to decimal:

$152_8 = 1 \times 8^2 + 5 \times 8^1 + 2 \times 8^0 = 106_{10}$

but how do you convert from 106_{10} to octal?

repeated division

convert 106_{10} to octal:

	Q	R	
106 ÷ 8	13	$8 \times 0.25 = 2$	2
13 ÷ 8	1	$8 \times 0.625 = 5$	5
1 ÷ 8	0	1	1

keep dividing until Q = 0

write these digits in reverse order

answer: 152_8

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convert 58 to binary:

	Q	R	
58 ÷ 2	29	0	
29 ÷ 2	14	1	(0.5 × 2)

58 ÷ 2	29	0
29 ÷ 2	14	1 (0.5×2)
14 ÷ 2	7	0
7 ÷ 2	3	1
3 ÷ 2	1	1
1 ÷ 2	0	1

111010_2

Convert 17980 to hexadecimal:

	Q	R
17980 ÷ 16	1123	0.75×16 = 12 (C)
1123 ÷ 16	70	0.1875×16 = 3
70 ÷ 16	4	0.375×16 = 6
4 ÷ 16	0	4

$463C_{16}$

Convert 1792 to octal:

	Q	R
1792 ÷ 8	224	0
224 ÷ 8	28	0
28 ÷ 8	3	4
3 ÷ 8	0	3

3400_8

Convert 53710 to hexadecimal

	Q	R
53710 ÷ 16	3356	14 (E)

	Q	R	
$53710 \div 16$	3356	~~14~~	E
$3356 \div 16$	209	~~12~~	C
$209 \div 16$	13	1	
$13 \div 16$	0	~~13~~	D

DICE$_{16}$

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so what about non-integer numbers?

instead of dividing by the base repeatedly, we multiply:

example: convert 0.375_{10} to binary

multiply the number 0.375 by the new base (2) and split it into the integer part and the non-integer part

	integer	non-integer
$0.375 \times 2 =$	0	+ 0.75
$0.75 \times 2 =$	1	+ 0.5
$0.5 \times 2 =$	1	+ 0

once this is zero, you can stop

now take the integer part

0 1

$0.375 = 0.011_2$

example: convert 0.8125 to binary

		int		non-int
0.8125×2	=	1	+	0.625
0.625×2	=	1	+	0.25
0.25×2	=	0	+	0.5
0.5×2	=	1	+	0

0.1101_2

example: convert 0.734375 to hexadecimal

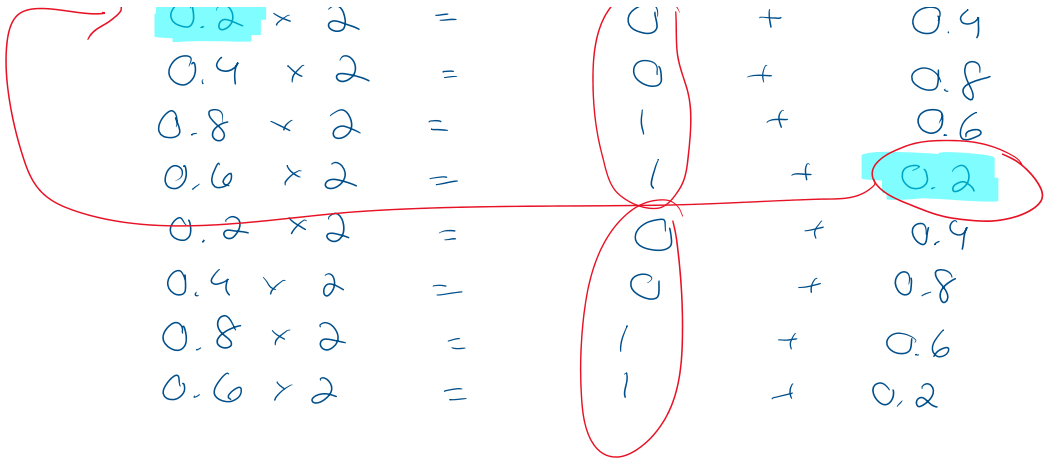
		int		non-int
0.734375×16	=	~~11~~ B	+	0.75
0.75×16	=	~~12~~ C	+	0

$0.BC_{16}$

an example with a twist:

convert 0.1 to binary

		int		non-int
0.1×2	=	0	+	0.2
0.2×2	=	0	+	0.4
0.4×2	=	0	+	0.8

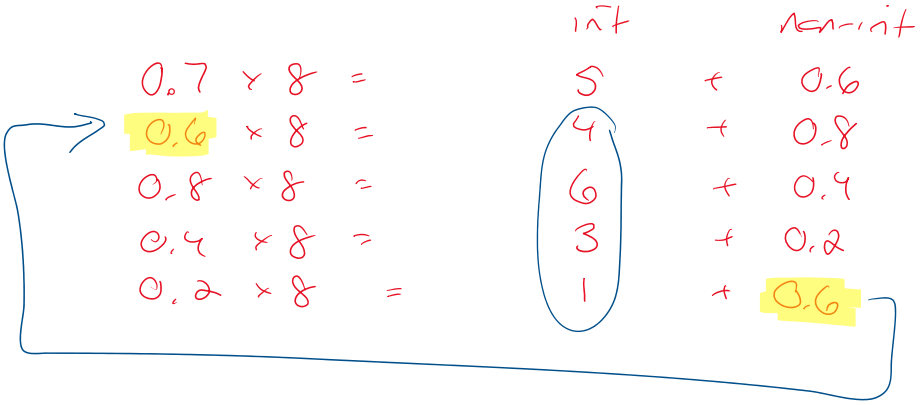


This is going to be a repeating pattern to the right of the radix point since the non-integer part will never go to zero

$$0.1_{10} = 0.0\ 0011\ 0011\ 0011\ \ldots$$

$$= 0.0\overline{0011}_2$$

convert 0.7 to octal:



$$0.5\overline{4631}_8$$

putting it all together:

do not round

convert 19.96875 to octal. Give an exact answer.

integer part

	Q	R
$19 \div 8$	2	3
$2 \div 8$	0	2

non-integer part

	int	non-int
$0.96875 \times 8 =$	7	+ 0.75
$0.75 \times 8 =$	6	+ 0

answer: 23.76_8

convert 52.5625 to base 4. Give an exact answer
Show your work.

	Q	R
$52 \div 4$	13	0
$13 \div 4$	3	1
$3 \div 4$	0	3

	int	non-int
$0.5625 \times 4 =$	2	+ 0.25
$0.25 \times 4 =$	1	+ 0

310.214