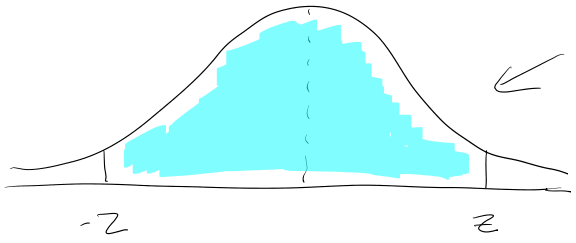
Section 10.2: Calculating Large Sample Confidence Intervals for the Mean

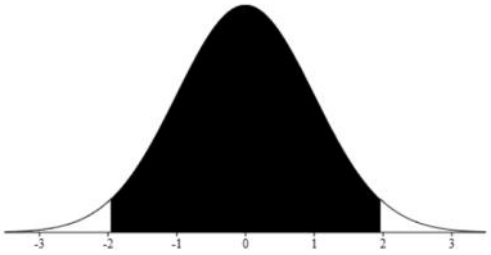
Tuesday, December 05, 2023 3:59 PM

The most common confidence interval is a 95% confidence interval



the shaded region should have area = 0.95

what value of z makes this so?



○ Area from a value (Use to compute p from Z)
◉ Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:

Area	0.95
Mean	0
SD	1

Results:

Recalculate

○ Above
○ Below
◉ Between -1.96 and 1.96
○ Outside

Screen clipping taken: 2023-12-05 4:06 PM

the online calculator gives $z = 1.96$ for 95% confidence

(not exactly 2, but very close to 2 by Empirical Rule)

other common intervals are 90%, 98%, and 99% and they also have z-scores associated with them

confidence level		z-score
90%	(0.90)	1.645
95%	(0.95)	1.96
98%	(0.98)	2.33
99%	(0.99)	2.575

for any other confidence level, use the online tool to find z

---

to construct a confidence interval, you need the following pieces of information:

- confidence level (like 95%), which gives you the associated z-score
- the sample mean $\bar{x}$
- standard deviation σ (for large samples, like we're doing here, can use sample standard deviation s instead)
- sample size n

then the population mean μ can be estimated by

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

← if σ is unknown, can use s for large samples

this whole term is called the margin of error

this whole term is called the margin of error (MOE)

note: this technique we are using only works for large samples in which $n \geq 30$

Section 10.2: cont'd

example: Forty students were asked how much time they studied the weekend before final exams. The mean was found to be 15.1 hours with a std dev of 6.5 hours

Construct confidence intervals for the mean time studied with

a) 90% confidence
b) 95% confidence
c) 99% confidence

what happens to the size of the interval as the confidence level increases?

answer: a) 90% confidence $z = 1.645$

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

$$= 15.1 \pm \frac{1.645(6.5)}{\sqrt{40}}$$

$$= 15.1 \pm 1.69063$$

too many decimal places

- in this class please round to the same precision (number of decimal places) as the mean (or to one more place)

$= 15.1 \pm 1.7$

CI = 13.4 to 16.8 hours

b) same calculation but with 95% $\rightarrow$ $z = 1.96$

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

$= 15.1 \pm 2.01437$

$= 15.1 \pm 2.0$

CI = 13.1 to 17.1 hours

c) 99% conf $\rightarrow$ $z = 2.575$

$\mu = 15.1 \pm 2.64693$

$= 15.1 \pm 2.6$

CI = 12.5 to 17.7 hours

As the confidence level increases, the width of the interval also increases.

---

what does increasing the sample size do?

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

MOE, margin of error

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

because n is in denominator, increasing n decreases the MOE and thus decreases the width of the interval

---

drawing conclusions based on confidence intervals

example: A study conducted by the doctors at a particular hospital involved monitoring a random sample of 75 surgery patients. The results showed that it took an average an amount of 3.2 cc of anesthetic A to put a patient to sleep, with a standard deviation of 0.4 cc.

However, the lastest medical research indicates that the average amount of anesthetic A needed is 3.0 cc.

a) Calculate a 99% confidence interval for the amount of anesthetic A needed in this hospital.

99% conf: $z = 2.575$

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

$$= 3.2 \pm \frac{2.575(0.4)}{\sqrt{75}}$$

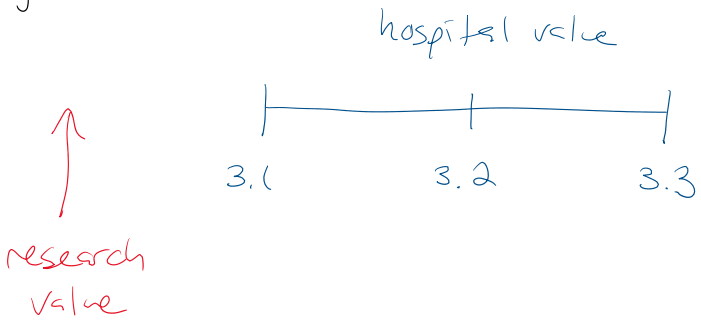
$$= 3.2 \pm 0.1189$$

$$= 3.2 \pm 0.1 \quad (\text{or } 3.2 \pm 0.12)$$

CI = 3.1 to 3.3 cc

b) Is there reason to believe that the hospital's value differs from the research value? Explain briefly.

answer:



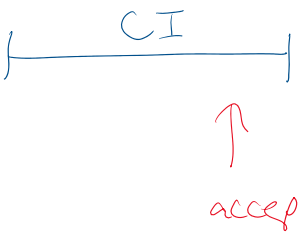
yes, because the research value lies outside the confidence interval

---

section 10.2: cont'd

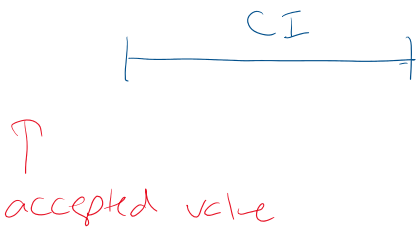
rule: comparing a confidence interval to an accepted value:

- if accepted value is with interval:



∴ agreement

- if accepted value is outside interval

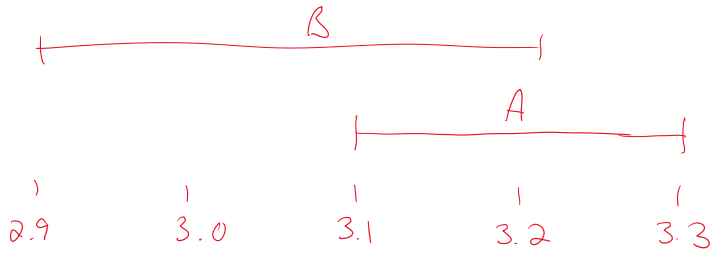


∴ disagreement

∴ means "therefore"

c) A similar study shows that the confidence interval for the amount of anesthetic B is 2.9 to 3.2 cc. Is there reason to believe that the amounts of A and B needed at this hospital are different?

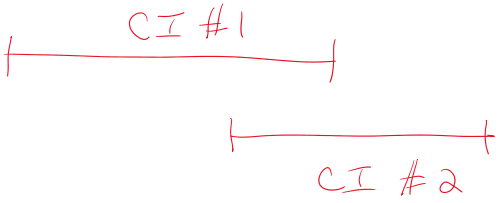
A: 3.1 to 3.3 cc
B: 2.9 to 3.2 cc



No, the intervals overlap.

rule: comparing two confidence intervals:

if they overlap:



∴ agreement

if they do not overlap:

∴ disagreement

---

how can you decrease the margin of error? $\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$

(1) increase sample size

↑ good scientific approach

(2) decrease the confidence level

↑ sad but practical approach if you have budget/time/resource restrictions

note: because the sample size n is under the square root, if we want to decrease the margin of error by a factor of 2, we need to increase the sample size by a factor of 4

one last thing:

THE MARGIN OF ERROR DOESN'T COVER ALL ERRORS

↑ the MOE reflects precision

- it does cover errors due to randomness in sampling

- it does not cover
  - non-representative samples
  - convenience samples
  - issues like non-response