

Section 2.1: Intro to Logic

Thursday, September 14, 2023 3:59 PM

logical proposition: a statement that is either true or false

examples:

- ① Python is a computer language.
- ② Bill Gates cofounded Microsoft.
- ③ The number seven is an even number.

non-examples:

- ④ Please put your books away.
- ⑤ Where is Sargta's office?
- ⑥ He is six feet tall.
who's he?

if the statement contains a variable (he) and that variable is undefined, then not a proposition

but "Paul is Pat's neighbour and he is six feet tall."

is a proposition

notation: use lower-case letters

p, q, r (and s, t)

example: let $p =$ "Pat drinks coffee"

operators:

"not" - negation

for p , the negation can be written as:

$\sim p$
 $\neg p$

}

used in symbolic logic
- advantage of using a
tilde (\sim) is that
it appears on a standard
keyboard

$!p$

often used in computing
("being p ")

\bar{p}

we'll see this in Boolean
algebra

p'

" p prime"

when are two propositions negations of each other?

- when only one of them can be true
at a time
(can't both be true, and can't both
be false)

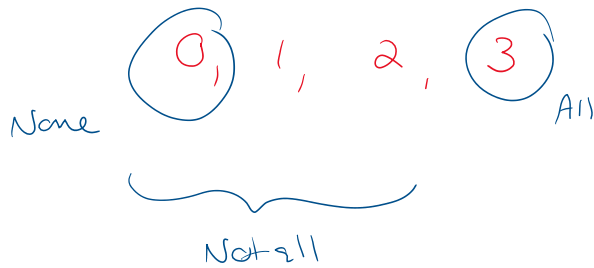
- there is no overlap and the two
propositions between them cover
all possibilities

the negation of "All are ..." or "Everyone is ..."

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"At least one person is not ..."

example: suppose I have three coins
how many are quarters?



None \neq Not all

↑
"at least one coin is
not a quarter"

because "not all" includes the cases
of 1 quarter and 2 quarters
while none does not

logical connectives:

"and" (conjunction) joins two propositions

notation: $p \wedge q$

$p \wedge q$ is true when both of p and q are true

$p \wedge q$ is false when at least one of p and q is false

(one or the other or both false)

other notations:

in computing, see $p \& \& q$ (C++/Java/JavaScript)

in English, there are other words that we use that are logically equivalent to "and" but sound more natural in sentences

Sanjay ate dinner and not dessert.

Sanjay ate dinner but not dessert.

but the second sentence sounds better than the first

"or" (inclusive disjunction)

notation: $p \vee q$

$p \vee q$ is true when at least one of p and q is true
(one or the other or both)

$p \vee q$ is false when both p and q are false

other notation: in computing, $p \parallel q$

"exclusive or" (exclusive disjunction)

notation: $p \oplus q$ \leftarrow I will use
 $p \text{ XOR } q$ \leftarrow frequently used
 $p \vee q$ in computing

(unfortunately, Java uses \wedge for exclusive or)

$p \oplus q$ is true when only one of p and q is true
(one or the other but not both)

problem: in English, the word "or" can mean either the "inclusive or" or the "exclusive or" and we tell the difference by context

Section 2.1: cont'd 2023/09/15

order of operations (ooo)

when you are doing arithmetic, to evaluate the expression

$$2 + 3 \cdot 4$$

you need to know the "order of operations": which comes first, adding (+) or multiplying (\cdot)?

in the same way, there is an order of

in the same way, there is an order of operations for logic:

"not" is done first

"and" is next

"or" is last

and brackets are used to override

-you can think of "not" like exponents
"and" like multiplication
"or" like addition

(more on this later)

so $p \vee q \wedge r$ is the same as $p \vee (q \wedge r)$
p or q and r

and $\sim p \vee q$ is the same as $(\sim p) \vee q$

note: if you wanted to negate $p \vee q$, use brackets:

$$\sim(p \vee q)$$

so brackets are used to override the default order of operations, just like in arithmetic:

$$(2+3) \cdot 4$$

↑

brackets mean addition first

examples: what is the order of operations?

- a) $\sim p \vee q$ not, then or
- b) $\sim(p \vee q)$ or, then not
- c) $p \vee \sim q \wedge r$ not, then and, then or
- d) $(p \vee \sim q) \wedge r$ not, then or, then and
- e) $p \vee \sim(q \wedge r)$ and, then not, then or