Section 2.1: Intro to Logic

Thursday, September 14, 2023 3:59 PM

logical proposition: a statement that is either true or false

examples:

1. Python is a computer language.
2. Bill Gates cofounded Microsoft.
3. The number seven is an even number.

non-examples:

4. Please put your books away.
5. Where is Sangita's office?
6. He is six feet tall.

who's he?

if the statement contains a variable (he) <u>and</u> that variable is undefined, then <u>not</u> a proposition

<u>but</u> "Paul is Pat's neighbor and he is six feet tall."

<u>is</u> a proposition

notation: use lower-case letters

p, q, r (and s, t)

example: let p = "Pat drinks coffee"

operators:

"not" - negation

for p, the negation can be written as:

$\sim p$
$\neg p$
} used in symbolic logic - advantage of using a tilde ($\sim$) is that it appears on a standard keyboard

$!p$ often used in computing ("bang p")

$\bar{p}$ we'll see this in Boolean algebra

p' "p prime"

when are two propositions negations of each other?

- when only one of them can be true at a time (can't both be true, and can't both be false)

- there is no overlap and the two propositions between them cover all possibilities

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the negation of "All are..." or "Everyone is..."

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"At least one person is <u>not</u> ..."

example: suppose I have three coins

how many are quarters?

None $\underbrace{0, \ 1, \ 2}_{\text{Not all}}, \ 3$ All

None $\neq$ Not all

$\uparrow$

"at least one coin is not a quarter"

because "not all" includes the cases of 1 quarter and 2 quarters

while none does not

logical connectives:

"and" (conjunction) joins two propositions

notation: $p \wedge q$

$p \wedge q$ is true when <u>both</u> of p and q are true

$p \wedge q$ is false when at least one of p and q is false

(one or the other or both false)

other notations:

in computing, see $p \&\& q$ (C++/Java/Javascript)

in English, there are other words that we use that are logically equivalent to "and" but sound more natural in sentences

Sanjay ate dinner <u>and</u> not dessert.

Sanjay ate dinner <u>but</u> not dessert.

but the second sentence sounds better than the first

"or" (inclusive disjunction)

notation: $p \vee q$

$p \vee q$ is true when <u>at least one</u> of p and q is true

(one or the other or both)

$p \vee q$ is false when both p and q are false

other notation: in computing, $p || q$

"exclusive or" (exclusive disjunction)

notation: $p \oplus q$ ← I will use

p XOR q ← frequently used in computing

$p \veebar q$

(unfortunately, Java uses $\wedge$ for exclusive or)

$p \oplus q$ is true when only one of p and q is true
(one or the other but not both)

problem: in English, the word "or" can mean either the "inclusive or" or the "exclusive or" and we tell the difference by context

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Section 2.1: cont'd 2023/09/15

order of operations (ooo)

when you are doing arithmetic, to evaluate the expression

$$2 + 3 \cdot 4$$

you need to know the "order of operations": which comes first, adding ($+$) or multiplying ($\cdot$)?

in the same way, there is an order of

in the same way, there is an order of operations for logic:

"not" is done first
"and" is next
"or" is last

and brackets are used to override

- you can think of "not" like exponents
"and" like multiplication
"or" like addition

(more on this later)

so $p \vee q \wedge r$ is the same as $p \vee (q \wedge r)$
p or q and r

and $\sim p \vee q$ is the same as $(\sim p) \vee q$

note: if you wanted to negate $p \vee q$, use brackets:

$$\sim (p \vee q)$$

so brackets are used to override the default order of operations just like in arithmetic:

$$(2+3) \cdot 4$$

brackets mean addition first

examples: what is the order of operations?

a) $\sim p \vee q$ not, then or

b) $\sim(p \vee q)$ or, then not

c) $p \vee \sim q \wedge r$ not, then and, then or

d) $(p \vee \sim q) \wedge r$ not, then or, then and

e) $p \vee \sim(q \wedge r)$ and, then not, then or