Section 2.5: The Laws of Logic (LoL)

Thursday, September 21, 2023 4:24 PM

there are connections between logic and Boolean algebra:

logic	$p \wedge q$	$p \vee q$	$\sim p$	F false	T true
Boolean algebra	AB	$A+B$	$\overline{A}$	0	1

we can show that

$p \wedge 1 \Leftrightarrow p$ can prove using truth table

p	1	$p \wedge 1$
0	1	0
1	1	1

similarly $A \cdot 1 = A$

these statements are true for all possible variables ($q \wedge 1 \Leftrightarrow q$, $\sim r \wedge 1 \Leftrightarrow \sim r$), so we call them laws. In particular, this one is called an identity law.

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identity laws: there are four of them:

$$p \wedge 1 \Leftrightarrow p$$
$$p \vee 1 \Leftrightarrow 1$$
$$p \wedge 0 \Leftrightarrow 0$$
$$p \vee 0 \Leftrightarrow p$$

but note that these statements are true for all possible variables.

So $p \vee 0 \iff p$, $q \vee 0 \iff q$

$\sim r \vee 0 \iff \sim r$

☺ $\vee\ 0 \iff$ ☺

$\sim(p \wedge q) \vee 0 \iff \sim(p \wedge q)$

why do we care?

example: simplify the following using the laws of logic

- use only one law of logic per line
- state the name of the law you are using

$$(p \wedge 0) \vee (q \wedge 1)$$

$0 \vee q$ identity

q "

idempotent

$$p \wedge p \iff p$$

$$p \vee p \iff p$$

which means that $\sim r \wedge \sim r \iff \sim r$

$\sim(p \wedge r) \vee \sim(p \wedge r) \iff \sim(p \wedge r)$

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digression (will not be tested)

idempotency in computing:

an operation is idempotent if an identical command can be made many times with the same effect as if it is made once

HTTP: this protocol contains a definition for idempotent

examples: elevator call buttons
crosswalk buttons

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Complement:

$$\sim(\sim p) \Leftrightarrow p$$

$$p \wedge \sim p \Leftrightarrow 0$$

$$p \vee \sim p \Leftrightarrow 1$$

examples: $q \wedge \sim q \Leftrightarrow 0$

$$ABC + \overline{ABC} = 1$$

note: for MATH 156, you may <u>omit</u> explicitly writing the commutative and associative laws as a separate step

example: $0 \vee p$

nitpicker solution (has extra steps) $p \vee 0$ commutative nitpicker solution (has extra steps that you do not need) $\quad p \vee 0 \quad$ commutative

$p \quad$ identity

totally acceptable MATH 156 solution $\quad p \quad$ identity

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example: simplify using the LOL

$$(\sim p \vee 0) \wedge (q \vee \sim q) \wedge (1 \vee r)$$

$\sim p \wedge (q \vee \sim q) \wedge 1 \quad$ identity

$\sim p \wedge 1 \wedge 1 \quad$ complement

$(\sim p \wedge 1) \wedge 1 \quad$ associative (can skip)

$\sim p \wedge 1 \quad$ identity

$\sim p \quad$ "

example: $(p \wedge \sim p) \vee (p \vee \sim p)$

$0 \vee 1 \quad$ complement

$1 \quad$ either $\begin{cases} \text{identity} \\ \text{complement} \\ \text{definition of "or"} \end{cases}$

simplify (brain tease)

$\sim(p \vee (q \wedge \sim r)) \quad \wedge \quad (p \vee (q \wedge \sim r))$

$0 \qquad$ complement

simplify $\qquad A(\bar{B}B) + B(A + \bar{A})$

$A \cdot 0 \quad + \quad B \cdot 1 \qquad$ complement

$0 \quad + \quad B \qquad$ identity

$B \qquad$ "

write a simplified expression for each of the following and identify which law you've used

1. $\sim r \wedge \sim r \Leftrightarrow \quad \sim r \qquad$ idempotent
2. $ABC + ABC = \quad ABC \qquad$ "
3. $\overline{ABC} + ABC = \quad 1 \qquad$ complement
4. $0 + \bar{B} = \quad \bar{B} \qquad$ identity

summary:

identity laws - involve ones and zeros

idempotent - involve a variable and/or itself

complement - involves negation

$\left.\begin{array}{l}\text{commutative} \\ \text{associative}\end{array}\right\}$ can skip