Section 2.6: More COL

Tuesday, September 26, 2023   3:42 PM

De Morgan's Laws:

$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{A+B} = \overline{A}\,\overline{B}$$

examples: use DeMorgan's to rewrite:

1. $\overline{B+C} = \overline{B}\,\overline{C}$
2. $\overline{\overline{B}+C} = B\,\overline{C}$
3. $\overline{\overline{B}+\overline{C}} = BC$
4. $\overline{\text{☺} + \text{☹}} = \overline{\text{☺}}\;\overline{\text{☹}}$
5. $\overline{A\overline{C}} = \overline{A} + C$
6. $\overline{A}C = \overline{A+\overline{C}}$

when would you see this in code?

if (x == 5 or y == 2) then do A
else do B

under what conditions does action B occur?

=> when (x == 5 or y == 2) is FALSE

in other words, when x != 5 AND y != 2

in other words, when $x != 5$ **AND** $y != 2$

Distributive:

$$A(B+C) = AB + AC$$

$$A + BC = (A+B)(A+C)$$

example: rewrite the following using the distributive law

1. $\overline{C}(A+C) = \overline{C}A + \overline{C}C$
2. $(A+B)(A+\overline{B}) = A + B\overline{B}$
3. $\overline{B} + \overline{A}\overline{C} = (\overline{B}+\overline{A})(\overline{B}+\overline{C})$
4. $\overline{AB}(B+\overline{C}) = \overline{AB}B + \overline{AB}\,\overline{C}$
5. $A\overline{C} + \overline{A}\,\overline{C} = \overline{C}(A+\overline{A})$

Absorption:

$$A(A+B) = A$$

$$A(\overline{A}+B) = AB$$

$$A + AB = A$$

$$A + \overline{A}B = A+B$$

examples: Use absorption laws to rewrite the following:

1. $\overline{B} + \overline{B}\,\overline{C} = \overline{B}$

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2. $\overline{C}(C + A) = \overline{C}A$

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3. $\overline{C}(\overline{C} + A) = \overline{C}$

4. $AB + ABC = AB$

5. $\overline{AB} + \overline{AB}C = \overline{AB} + C$ $\qquad A + \overline{A}B = A + B$

Simplify the following using the Laws of Logic (LOL)
- only one law of logic per line
- state the name of law you are using.

$$(\sim p \vee \sim q) \wedge (p \vee \sim q)$$

$$\boxed{(p \vee q) \wedge (p \vee r) \Leftrightarrow p \vee (q \wedge r)}$$

$\sim q \vee (\sim p \wedge p)$ distrib

$\sim q \vee 0$ complement

$\sim q$ identity

simplify using the LOL

$$AB(\overline{A} + \overline{B})$$

method #1: $AB\bar{A} + AB\bar{B}$ distributive

$B \cdot 0 + A \cdot 0$ complement

$0 + 0$ identity

0 idempotent / identity / definition of "or"

$AB(\bar{A} + \bar{B})$

method #2 $AB\ \overline{AB}$ DeMorgan's

0 Complement

$AB\ (\bar{A} + \bar{B})$

method #3 $AB\bar{A}$ absorption

$B \cdot 0$ complement

0 identity

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prove: $\overline{B \cdot 0} = \bar{A} + \overline{\bar{A}\bar{B}}$

$\bar{0} = \bar{A} + \overline{\bar{A}\bar{B}}$ identity

$1 = \bar{A} + \overline{\bar{A}\bar{B}}$ definition of "not"

$$= \bar{A} + A + B$$ DeMorgan's

$$= 1 + B$$ complement

$$= 1$$ identity

QED

simplify: $\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$

method #1 $\bar{B}\bar{A} + \bar{A}$ absorption

$\bar{A}$ "

$$\bar{B}(\bar{A} + B) + \bar{A}(\bar{A} + B)$$

method #2: $\bar{B}\bar{A} + \bar{B}B + \bar{A}\bar{A} + \bar{A}B$ distrib

$\bar{B}\bar{A} + 0 + \bar{A}\bar{A} + \bar{A}B$ complement

$\bar{B}\bar{A} + \bar{A}\bar{A} + \bar{A}B$ identity

$\bar{B}\bar{A} + \bar{A} + \bar{A}B$ idempotent

$\swarrow$ $\searrow$

$\bar{A} + \bar{A}(\bar{B} + B)$ distrib

$\bar{A} + \bar{A} \cdot 1$ complement

$\bar{A} + \bar{A}$ identity

$\bar{A}$ idempotent

$\bar{B}\bar{A} + \bar{A}$ absorption

$\bar{A}$ "

$$\bar{B}(\bar{A}+B)+\bar{A}(\bar{A}+B)$$

method # 3

$(\bar{A}+B)(\bar{B}+\bar{A})$ distrib

$\bar{A}+B\bar{B}$ "

$\bar{A}+0$ complement

$\bar{A}$ identity

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prove $A+\overline{A+B} = A+\bar{B}C+\bar{B}\bar{C}$

$A+\overline{A+B} = A+\bar{B}(C+\bar{C})$ distrib

$= A+\bar{B}\cdot 1$ complement

$= A+\bar{B}$ identity

$A+\bar{A}\bar{B} =$ DeMorgan's

$A+\bar{B} =$ " absorption