

Section 2.7: The Conditional

Tuesday, October 03, 2023 3:34 PM

conditional:

$$p \rightarrow q$$

"p implies q"

"if p, then q"

example: "If I live in Scanich, then I live in BC."

example: If Barney is a dog, then Barney has four legs. ← true

Answer the following questions with "yes," "no," or "maybe."

- a) Barney is a dog. Does he have four legs? Y
 - b) Barney is not a dog. Does he have four legs? M
 - c) Barney has four legs. Is he a dog? M
 - d) Barney does not have four legs. Is he a dog? N
-

example: The following statement is true:

If ^p Snarks are Boogums, then the
cannot have p true and q false Bellman is incorrect.
q

cannot have p true
and q false

Bellman is incorrect
 q

Which of the following cannot occur?

- a) Snacks are Boojums and the Bellman is incorrect.
- b) Snacks are not Boojums and the Bellman is incorrect.
- c) Snacks are not Boojums and the Bellman is correct.
- d) Snacks are Boojums and the Bellman is correct.

example: If Pet sleeps in, then she will be late for class.

For this conditional $p \rightarrow q$, write the converse $q \rightarrow p$.
Is the converse logically equivalent to the conditional?

converse $q \rightarrow p$

If Pet is late for class, then she slept in.

logically equivalent? No!

so, how can we prove A? truth table

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	1
0	1	1	0
1	0	0	1

0	1	1	0
1	0	0	1
1	1	1	1

$p \rightarrow q$ is not logically equivalent to $q \rightarrow p$

$$(p \rightarrow q) \not\equiv (q \rightarrow p)$$

example: consider the conditional $p \rightarrow q$. Is it logically equivalent to the contrapositive $\sim q \rightarrow \sim p$? Justify your answer using a truth table.

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If $p \rightarrow q$ is true,
can't have
first one true
and second one
↓ false

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \rightarrow \sim p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

Yes

or, if you insist,

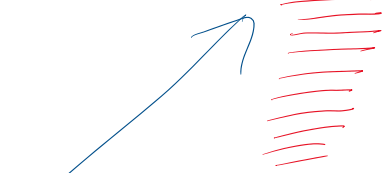
$$(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$$

example: write the contrapositive $\neg q \rightarrow \neg p$ for the
the following conditional $p \rightarrow q$

If Barney is a poodle or a spaniel, then
Barney is a dog.

answer: If Barney is not a dog, then
Barney is not a poodle AND not a spaniel.

stealth DeMorgan's



why?

$\overline{\text{poodle or spaniel}} \Leftrightarrow \overline{\text{poodle}} \text{ and } \overline{\text{spaniel}}$

another perfectly acceptable answer would be:

If Barney is not a dog, then Barney is
neither a poodle nor a spaniel.

why?

NOR = "not or"

digression: will not be tested

$p \text{ NOR } q$ can be written $p \downarrow q$
 $p \text{ NAND } q$ $p \uparrow q$

$$p \text{ NOR } q \Leftrightarrow \sim(p \vee q)$$

what about the inverse $\sim p \Rightarrow \sim q$?

	conditional	$p \rightarrow q$	
	converse	$q \rightarrow p$	
	contrapositive	$\sim q \rightarrow \sim p$	
	inverse	$\sim p \rightarrow \sim q$	
equivalent			equivalent

the "or" form of the conditional:

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

$$(p \rightarrow q) \Leftrightarrow (\sim p \vee q)$$

example: for the following conditional $p \rightarrow q$, rewrite it in the "or" form, $\sim p \vee q$

If Pat sleeps in, then she will be late for class.

answer: Pat didn't sleep in or she was late for class.

discussion: why do we care?

pseudo code:

```
if  $x > 3$  then  $y = 4$   
print  $y$ 
```

question: if the output is "4", was $x > 3$?

consider:

$x = 5$

$y = 7$

if $x > 3$ then $y = 4$

print y

output = 4

$x = 2$

$y = 9$

if $x > 3$ then $y = 4$

print y

output = 4

answer to question
is MAYBE