Section 2.7: The Conditional

Tuesday, October 03, 2023 3:34 PM

conditional:

$$p \to q$$

"p implies q"
"if p, then q"

example: "If I live in Saanich, then I live in BC."

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example: If Barney is a dog, then Barney has four legs. ← true

Answer the following questions with "yes," "no", or "maybe".

a) Barney is a dog. Does he have four legs? Y

b) Barney is not a dog. Does he have four legs? M

c) Barney has four legs. Is he a dog? M

d) Barney does not have four legs. Is he a dog? N

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example: The following statement is true:

If Snarks are Boojums (p), then the Bellman is incorrect (q).

Cannot have p true and q false

Cannot have p true and q false

Bellman is incorrect. q

Which of the following cannot occur?

a) Snarks are Boojums (p true) and the Bellman is incorrect.

b) Snarks are not Boojums and the Bellman is incorrect.

c) Snarks are not Boojums and the Bellman is correct. (q false)

d) Snarks are Boojums (p true) and the Bellman is correct. (q false)

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example: If Pat sleeps in, then she will be late for class.

For this conditional $p \to q$, write the converse $q \to p$. Is the converse logically equivalent to the conditional?

converse $q \to p$

If Pat is late for class, then she slept in.

logically equivalent? No!

So, how can we prove it? truth table

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1

0	1	1	0
1	0	0	1
1	1	1	1

$p \to q$ is <u>not</u> logically equivalent to $q \to p$

$$(p \to q) \not\Leftrightarrow (q \to p)$$

example: consider the conditional $p \to q$. Is it logically equivalent to the contrapositive $\sim q \to \sim p$? Justify your answer using a truth table.

Section 2.2 cont'd
2023/10/04

if $p \to q$ is true,
can't have
first one true
and second one
false

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

$\boxed{\text{Yes}}$

or, if you insist,

$$(p \to q) \Leftrightarrow (\sim q \to \sim p)$$

example: write the contrapositive $\sim q \to \sim p$ for the the following conditional $p \to q$

If Barney is a poodle or a spaniel, then Barney is a dog.

answer: If Barney is not a dog, then

Barney is not a poodle AND not a spaniel.

stealth DeMorgan's

why?

$\overline{\text{poodle or spaniel}} \iff \overline{\text{poodle}}$ and $\overline{\text{spaniel}}$

another perfectly acceptable answer would be:

If Barney is not a dog, then Barney is

neither a poodle nor a spaniel.

why? NOR = "not or"

digression: will not be tested

p NOR q can be written $p \downarrow q$

p NAND q $p \uparrow q$

p NOR $q \Leftrightarrow \sim(p \vee q)$

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what about the inverse $\sim p \to \sim q$?

	conditional	$p \to q$
	converse	$q \to p$
equivalent	contrapositive	$\sim q \to \sim p$
	inverse	$\sim p \to \sim q$

equivalent

the "or" form of the conditional:

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

same

$(p \to q) \Leftrightarrow (\sim p \vee q)$

example: for the following conditional $p \to q$, rewrite it in the "or" form, $\sim p \vee q$.

If Pat sleeps in, then she will be late for class.

answer: Pat didn't sleep in or she was late for class.

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digression: why do we care?

pseudo code:

if $x > 3$ then $y = 4$
print y

question: if the output is "4", was $x > 3$?

consider:

$x = 5$ $y = 7$ if $x > 3$ then $y = 4$ print y output = 4	$x = 2$ $y = 4$ if $x > 3$ then $y = 4$ print y output = 4

answer to question
is MAYBE