

Section 2.8: The Biconditional

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biconditional:

"If you get 90% or higher in Math 156,
then you will get an A+
and vice versa."

Section 2.8 cont'd
2023/10/05

"If and only if you get 90% or higher in
Math 156, then you will get an A+."

what does it mean?

if p , then q and if q , then p

or if p , then q and if $\sim p$, then $\sim q$.

or "either p and q are both true or
they are both false"

notation:

$$p \leftrightarrow q$$

"if and only if p , then q " (iff p , then q)

"if p , then q and vice versa"

example: consider the following conditional statements.
Do they still make sense when written
as a biconditional?

① If you eat at Joe's, then you will have a good meal.

No

② If two lines are perpendicular, then they meet at a right angle.

Yes

③ If today is Wednesday, then tomorrow is Thursday.

Yes

④ If the car battery is drained, then the car will not start.

No

truth table:

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

example: Is the biconditional $p \Leftrightarrow q$ logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$?
Use a truth table to justify your answer.

p	q	$p \Leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

Yes

or, if you insist, $(p \Leftrightarrow q) \Leftrightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$

example: The following statement is true:

If and only if ^p Snarks are Boojums,
then the _q Bellman is incorrect.

- a) Snarks are Boojums. Is the Bellman correct? No
p is true so *q is also true*
- b) Snarks are not Boojums. Is the Bellman correct? Yes
p is false so *q is also false*
- c) The Bellman is correct. Are Snarks Boojums? No
q is false so *p is false*
- d) The Bellman is incorrect. Are Snarks Boojums? Yes
q is true so *p is true*