Section 3.2: Arithmetic Sequences and Series

Thursday, October 12, 2023 3:58 PM

examples:

pattern?

1. $2, 5, 8, \ldots$ — add 3
2. $0.4, 0.5, 0.6, \ldots$ — add 0.1
3. $5, -5, -15, \ldots, -205$ — add -10
4. $-1, -3/2, -2, -5/2, \ldots, -10$ — add $-1/2$

1 and 2 are infinite
3 and 4 are finite

arithmetic sequence $\equiv$ a sequence in which you find the next term by adding a constant to the previous term

common difference d

why? subtract any term from the next term to find it

recursive form for arithmetic sequences:

example: write a recursive formula for the sequence

$0.4, 0.5, 0.6, \ldots$

answer:

$$\begin{cases} a_0 = 0.4 \\ a_n = a_{n-1} + 0.1 \quad \text{for } n \geq 1 \end{cases}$$

(or $n > 0$)

$$\begin{cases} a_n = a_{n-1} + 0.1 & \text{for } n \geq 1 \\ & \text{(or } n > 0) \end{cases}$$

in general:

a_m = <insert first term here>

$a_n = a_{n-1} + d \quad$ for $n > m$

or $n \geq m+1$

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general formula: $2, 5, 8, \ldots$

1	2	3	4		n	
2,	5,	8,	11,	...	a_n,	...
2,	$2+3$,	$2+2\cdot 3$,	$2+3\cdot 3$,	...	$2+(n-1)\cdot 3$	

so for this particular example

$$a_n = 2 + (n-1)\cdot 3$$

and in general,

$$\boxed{a_n = a_m + (n-m)\cdot d \quad \text{for } n \geq m}$$

note: to find the <u>simplified</u> general formula for $2, 5, 8, \ldots$

we simplify

$$a_n = 2 + (n-1)3$$
$$= 2 + 3n - 3$$
$$\boxed{a_n = 3n - 1 \quad \text{for } n \geq 1}$$

example: find a general formula for the sequence $5, -5, -15, \ldots$

simplify your answer and specify your starting index simplify your answer and specify your starting index

answer: this sequence is arithmetic

let's use $m=0$ as starting index

$a_0 = 5$
$d = -10$

$a_n = a_m + (n-m)d$

$a_n = 5 + (n-0)(-10)$

$$\boxed{a_n = 5 - 10n \quad \text{for } n \geq 0}$$

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the advantage of the general formula is that computing terms is more straightforward than for the recursive formula

- to find the 10 000th term in a recursive formula, you need to know the 9999th term, which requires the 9998th term, and so on
- to find the 10000th term in a general series, you need to just plug in for n

example: for the arithmetic sequence in which the first term is 2 and the fiftieth term is 394, what is the common difference?

answer: $a_n = a_m + (n-m)d$

$394 = 2 + (50-1)d$

$392 = 49d$

$\boxed{d = 8}$

The common difference is 8

An arithmetic sequence has the seventy-fifth term equal to -437. If the common difference is -6, what is the first term?

answer: let $m=1$

$a_n = a_m + (n-m)d$

$-437 = a_1 + (75-1)(-6)$

$-437 = a_1 - 444$

$\boxed{a_1 = 7}$

let $m=0$

$a_n = a_m + (n-m)d$

$-437 = a_0 + (74-0)(-6)$

etc

The first term is 7

example: consider the sequence

$$18, \overset{+12}{30}, \overset{+12}{42}, \ldots, 582$$

How many terms in the sequence?

arithmetic with $d = 12$

$a_n = a_m + (n-m)d$

$582 = 18 + (n-1)12$

$564 = 12(n-1)$

$47 = n-1$

$n = 48$

$\boxed{\text{There are 48 terms}}$

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arithmetic series

$2 + 5 + 8 + \ldots$

for this series, calculate S_8 ess

answer: $S_8 = \underbrace{2 + \overbrace{5 + \underbrace{8 + \overbrace{11 + 14} + 17}_{} + 20}^{25} + 23}_{25}$

$= 4 \times 25$

number of pairs

sum of each pair

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$$\boxed{S_k = \frac{k}{2}(a_m + a_n)}$$

$k = n - m + 1$

but what if k is odd?

$S_7 = 2 + \overbrace{5 + \underbrace{8 + 11 + 14} + 17}^{22} + 20$

1

$$S_7 = 2 + 5 + 8 + \underbrace{\overbrace{11}^{\text{half of }22}}_{} + 14 + 17 + 20$$

22

half of 22

$3\frac{1}{2}$ pairs whose sum is 22

$\to$ still works!

summary:

$$\boxed{\begin{aligned} S_k &= \frac{k}{2}(a_m + a_n) \\ S_k &= \frac{k}{2}\left(2a_m + (n-m)d\right) \end{aligned}}$$

$k = n - m + 1$

$a_n = a_m + (n-m)d$

example: find the sum of the first 50 terms of

$2 + 5 + 8 + \ldots$

answer: method #1: arithmetic with $d = 3$

$m = 1$
$n = 50$
$k = 50$

$\begin{bmatrix} m = 0 \\ n = 49 \\ k = 50 \end{bmatrix}$

$$\begin{aligned} S_k &= \frac{k}{2}\left(2a_m + (n-m)d\right) \\ &= \frac{50}{2}\left(2 \cdot 2 + (50-1) \cdot 3\right) \\ &= 3775 \end{aligned}$$

method #2: arithmetic with $d = 3$

$$S_k = \frac{k}{2}(a_m + a_n)$$
$$= \frac{50}{2}(2 + \quad)$$

but what's this?

$$a_n = a_m + (n-m)d$$
$$= 2 + (50-1)\cdot 3$$
$$= 149$$

$$= \frac{50}{2}(2+149)$$
$$= 3775$$

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evaluate:

$$\sum_{j=4}^{50}(6j-3) = (6\cdot4-3) + (6\cdot5-3) + (6\cdot6-3) + \dots (6\cdot50-3)$$

4, 5, 6, ..., 50

$$= 21 + 27 + 33 + \dots + 297$$

arithmetic with $d=6$

$$S_k = \frac{k}{2}(a_m + a_n)$$
$$= \frac{47}{2}(21+297)$$
$$= 7473$$

$k = n-m+1$
$= 50-4+1 = 47$

summary: arithmetic

$$\boxed{\begin{aligned} a_n &= a_m + (n-m)d \\ S_k &= \frac{k}{2}(a_m + a_n) \\ S_k &= \frac{k}{2}(2a_m + (n-m)d) \end{aligned}}$$

$\Leftarrow$ for $n \geq m$

} where $k = n-m+1$