Section: Geometric Sequences and Series

Tuesday, October 17, 2023 4:05 PM

examples:

pattern?

1. $7, 14, 28, 56, \ldots, 114688$ — mult by 2
2. $100, 20, 4, 4/5, \ldots$ — mult by $1/5$
3. $1/2, 1/4, 1/8, 1/16, \ldots, 1/256$ — mult by $1/2$
4. $24, -16, 32/3, -64/9, \ldots$ — mult by $-2/3$

but how do you find this?

take any term except the first and divide by previous

so $\frac{-16}{24} = \frac{-2\cdot\cancel{8}}{3\cdot\cancel{8}} = -\frac{2}{3}$ (or $-0.\overline{6}$)

and $\frac{32}{3} \div (-16) = \frac{\cancel{32}^{\,2}}{3}\cdot\frac{1}{\cancel{-16}} = \frac{-2}{3}$

geometric sequence $\equiv$ a sequence in which the next term is equal to the previous term multiplied by a constant

common ratio r

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recursive formula:

example: give a recursive formula for the example: give a recursive formula for the sequence

$$100, \quad 20, \quad 4, \quad 4/5, \quad \ldots$$

answer: $\begin{cases} a_1 = 100 \\ a_n = \frac{1}{5} a_{n-1} \end{cases}$ for $n > 1$, $n \geq 2$, $n = 2, 3, 4, \ldots$

any of these

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general formula:

example: find a general formula for $7, 14, 28, \ldots$

1	2	3	4		n	
$7,$	$14,$	$28,$	$56,$	$\ldots,$	$a_n,$	$\ldots$
$7,$	$7 \cdot 2,$	$7 \cdot 2^2,$	$7 \cdot 2^3,$		$7 \cdot 2^{n-1}$	

the general form for this particular sequence is

$$\boxed{a_n = 7 \cdot 2^{n-1} \quad \text{for } n \geq 1}$$

for geometric,

$$\boxed{a_n = a_m r^{n-m} \quad \text{for } n \geq m}$$

example: consider the sequence $5, 15, 45, \ldots$

find a) the twelfth term

b) the fiftieth term

answer: geometric with $r = 3$

$$a_n = a_m r^{n-m}$$

a) twelfth term: if $m = 1$, we want a_{12}

$$a_{12} = 5 \cdot 3^{12-1} = 5 \cdot 3^{11} = 885\,735$$

b) fiftieth term

$$a_{50} = 5 \cdot 3^{49} = 1.196 \times 10^{24}$$

example: give the general formula for

$$80, -160, 320, \ldots$$

answer: geometric with $r = -2$

$$a_n = a_m r^{n-m}$$

$$\boxed{a_n = 80(-2)^{n-1} \quad \text{for } n \geq 1}$$

note: $a_n = 80 - 2^{n-1}$ is not correct

$= 80 * -2^{n-1}$ is not correct either

why not? $-2^2 = -(2^2)$

$$= -4$$

$$(-2)^2 = (-2)(-2) = 4$$

in WeBWorK, write 80(-2)^(n-1)

geometric series: $7 + 14 + 28 + \ldots$

$$S_k = \frac{a_m(1-r^k)}{1-r}$$

sum of the first k terms

where a_m = first term, r = common ratio

$k \geq 1$ and $r \neq 1$

digression: will not be tested

why?

$$S_k = a_1 + a_2 + a_3 + \ldots + a_k$$

$$\left.\begin{aligned} S_k &= a_1 + a_1 r + a_1 r^2 + \ldots + a_1 r^{k-1} \\ -rS_k &= -a_1 r - a_1 r^2 - a_1 r^3 - \ldots - a_1 r^k \end{aligned}\right\} \text{add}$$

$$S_k - rS_k = a_1 - a_1 r^k$$

$$S_k(1-r) = a_1(1-r^k)$$

$$S_k = \frac{a_1(1-r^k)}{1-r} \quad \text{for } k \geq 1$$

example: for the series $2 + 10 + 50 + \ldots$

find the sum of the first

a) twelve terms
b) fifty terms

answer: geometric with $r = 5$

$$S_k = a_m \frac{(1 - r^k)}{1 - r}$$

so $$S_{12} = \frac{2(1 - 5^{12})}{1 - 5} = 122\ 070\ 312$$

or 1.22×10^8

$$S_{50} = \frac{2(1 - 5^{50})}{1 - 5} = 4.44 \times 10^{34}$$

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what about $1/4 + 1/16 + 1/64 + \ldots$?

how do we make this work?

$$S_k = \frac{a_m (1 - r^k)}{1 - r}$$

$$S_\infty = \frac{a_m (1 - r^\infty)}{1 - r}$$

but what is r^∞ ? $r = 1/4$

let's consider $\left(\frac{1}{4}\right)^n$

as $n \to \infty$, $\left(\frac{1}{4}\right)^n \to 0$

but <u>only</u> for

$-1 < r < 1$

or $|r| < 1$

then $S_\infty = \frac{a_m(1 - \cancel{r^\infty})}{1-r}$ $\xrightarrow{} 0$

$$\boxed{S_\infty = \frac{a_m}{1-r} \quad \text{for} \quad -1 < r < 1}$$

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so, back to our original question,

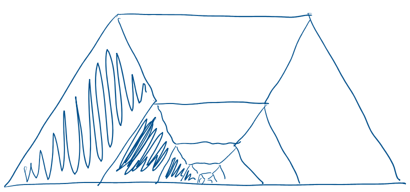
$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$

geometric with $r = \frac{1}{4}$

is $-1 < r < 1$? yes

$$S_\infty = \frac{a_m}{1-r}$$

$$= \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3} = \frac{1}{3}$$



evaluate: $24 - 16 + 32/3 - \ldots$

answer: geometric with $r = \frac{-16}{24} = \frac{-2 \cdot \cancel{8}}{-3 \cdot \cancel{8}} = \frac{-2}{3}$

is $-1 < r < 1$? yes

$$S_\infty = \frac{a_m}{1-r}$$

$$= \frac{24}{1-(-2/3)} = \frac{24}{5/3} = 24 \cdot \frac{3}{5} = \frac{72}{5} \text{ or } 14.4$$

evaluate: $12 + 18 + 27 + \ldots$

$$\cancel{S_\infty} = \cancel{\frac{a_m}{1-r}} = \cancel{\frac{12}{1-3/2}} = \cancel{\frac{12}{-1/2}} = \cancel{12 \times (-2)} = \cancel{-24}$$

is $-1 < r < 1$? no

S_∞ is undefined

S_∞ does not exist (DNE)

note: for series like

$12 + 18 + 27 + \ldots$, you could say

$S_\infty \to \infty$

$-12 - 18 - 27 - \ldots$, you could say $S_\infty \to -\infty$

but for alternating series with r negative
$12 - 18 + 27 - \ldots$

can only say DNE or undefined

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evaluate:

$$\sum_{j=0}^{\infty} 75\left(\frac{3}{5}\right)^j = \overset{0}{75} + \overset{1}{75\left(\frac{3}{5}\right)} + \overset{2}{75\left(\frac{3}{5}\right)^2}$$

$$= 75 + 45 + 27 + \ldots$$

geometric with $r = \frac{3}{5}$ (or 0.6)

$-1 < r < 1$? yes

$$S_\infty = \frac{a_m}{1-r} = \frac{75}{1 - 3/5} = \frac{75}{2/5} = 187.5$$

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digression: will not be tested

$$0.\overline{4} = 0.444444\ldots$$

$$= 0.4 + 0.04 + 0.004 + 0.0004 + \ldots$$

geometric with $r = 1/10$ $\quad -1 < r < 1$? yes

$$S_\infty = \frac{a_m}{1-r} = \frac{4/10}{1 - 1/10} = \frac{4/10}{9/10} = \frac{4}{10} \cdot \frac{10}{9} = \frac{4}{9}$$

$$S_\infty = \frac{a_m}{1-r} = \frac{4/10}{1-1/10} = \frac{4/10}{9/10} = \frac{4}{10}\cdot\frac{10}{9} = \frac{4}{9}$$

summary:

$$\boxed{a_n = a_m r^{n-m} \qquad S_k = \frac{a_m(1-r^k)}{1-r} \qquad S_\infty = \frac{a_m}{1-r}}$$

for $n \geq m$

$k = n-m+1$

$-1 < r < 1$