Section 4.1: Big O and Rates of Growth

Thursday, October 19, 2023 4:32 PM

example: suppose you want to own digital copies of a number of Star Trek episodes. You can either download them at one episode per hour or you can get overnight delivery from an online retailer (call it 24 hour delivery).

What method should you choose?

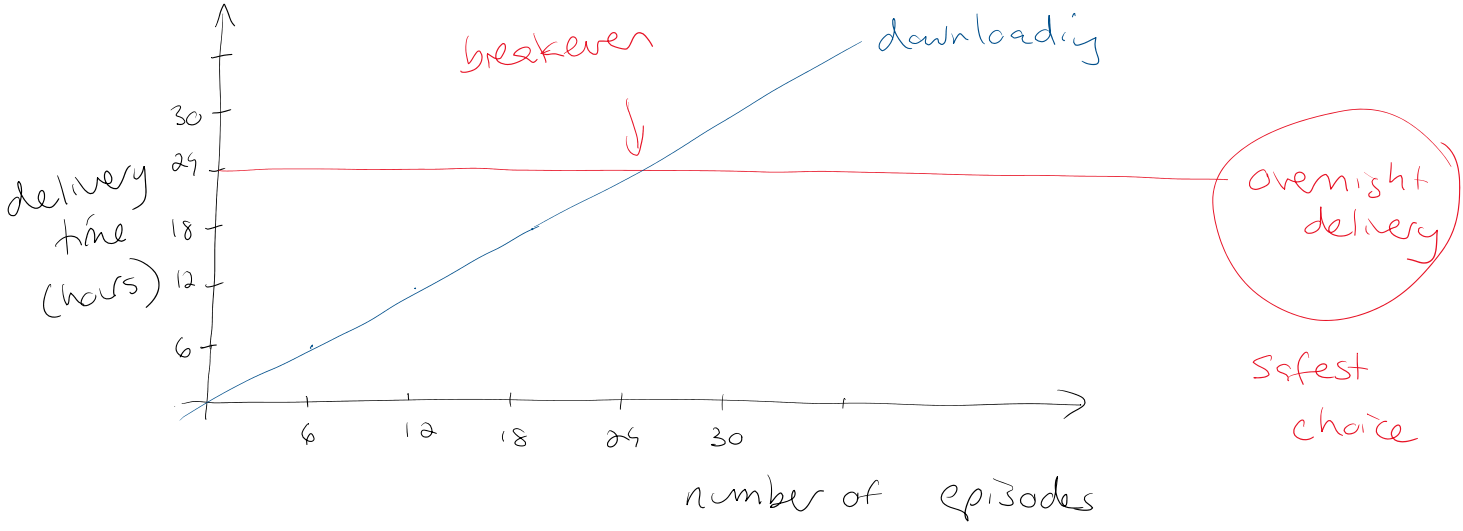
answer: scenarios: we want minimum delivery time

few episodes (< 24), downloading faster

breakeven ($= 24$), methods take same time

many episodes (> 24), overnight delivery

and if you don't know how many episodes are needed, then the "many episodes" scenario is the safest approach



number of steps required for algorithms

Consider adding two 5-digit numbers:

```
  12345
+ 12345
```

best case scenario
5 additions

worst case
5 additions plus
some "carry"s

Consider multiplying two 5-digit numbers

```
  12345
x 12345
  xxxxx
 xxxxx
xxxxx
 xxxxx
xxxxx
```

best case scenario:
25 multiplications
plus 9 column additions
and no carries

worst case
25 multiplications
plus 10 column additions
and lots of carries

The worrying part of multiplications is the fact that the number is the square of the number of digits

100 digits → 10000 multiplications!

ouch!

addition: if you double the number of digits, you approximately double the number of operations

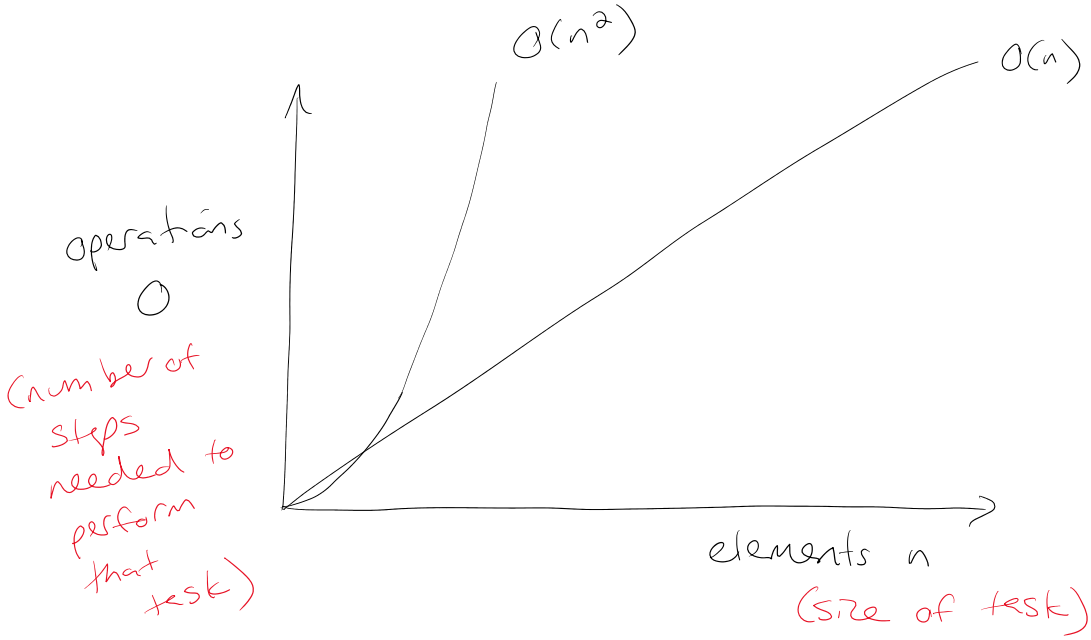
Big O notation: $O(n)$

"of order n"

multiplication: if you double the numbe of digits, you approximately quadruple the numbe of operations

$O(n^2)$

note: in Math 156, I will not ask you to analyse an algorithm and figure at Big O



note that even if the straight line had

Note that even if the straight line had a very steep slope, the $O(n^2)$ graph will cross it for sufficiently large values of n

→ Big O is only concerned with the number of operations for very large n

"worst case scenario"