Section 4.2: Factorial and Exponential Growth

Friday, October 20, 2023 11:32 AM

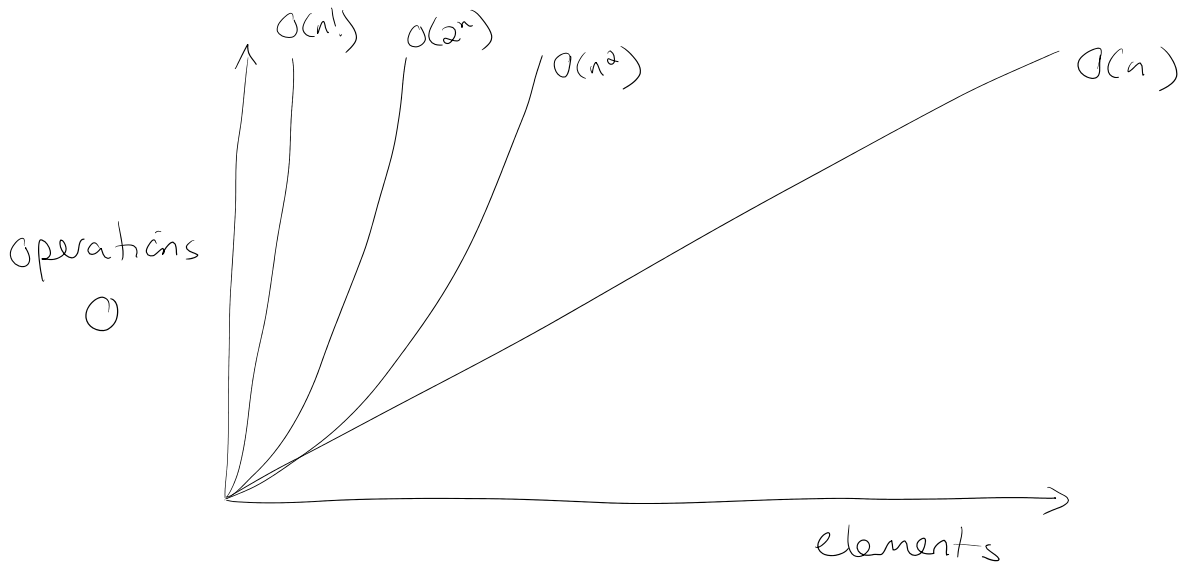
factorial: $n!$

$3! = 3 \cdot 2 \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$n! = n(n-1)(n-2) \dots \cdot 3 \cdot 2 \cdot 1$

n	polynomial n^2	exponential 2^n	factorial $n!$
1	1	2	1
2	4	4	2
3	9	8	6
4	16	16	24
5	25	32	120
10	100	1024	3 628 800
100	10000	1.267×10^{30}	9.33×10^{157}



what I will be testing you on is ranking the various orders

$O(n)$, $O(n^2)$, ...

so that you know the shape of each graph and are able to tell which one is more efficient as n gets large

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Big O for sums of different functions

what if your procedure requires $n^2 + 2n + 5$ steps for a task of size n? what is Big O?

n	n^2	$2n$	5	n^2+2n+5
1	1	2	5	8
10	100	20	5	125
100	10000	200	5	10205
1000	1000000	2000	5	1002005

as n gets large, the contributions to the total from $2n$ and 5 become very small in comparison to the contribution from the n^2 term

so for large n, $O(n^2+2n+5) \approx O(n^2)$

↑ approximately equal to

to find Big O for a sum of different functions:

- locate in the sum the term that grows the fastest
- remove any coefficients

- what's left is Big O

examples: consider procedures where the number of operations required for a task of size n is given below. Find Big O for each procedure

a) $9n + 5$ answer: $O(n)$

b) $2^n + n^2$ $O(2^n)$

c) $4^2 + 4!$ $O(1)$

d) $n! + 4!$ $O(n!)$

e) $3n(n+1) = 3n^2 + 3n$ $O(n^2)$

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digression: will not be tested

so where might you see these types of growth?

- you have a sorted list of size n, what is the highest value?

$O(1)$

- you have a loop from 1 to n (n = size of task)

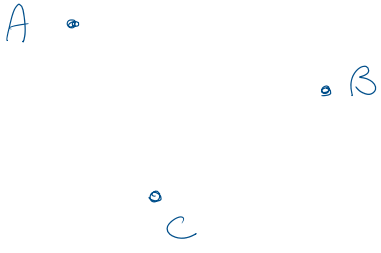
$O(n)$ - linear search

- you have two nested loops, each from

- you have two nested loops, each from 1 to n

$O(n^2)$ bubble sort

- traveling saleman problem



possible routes:

ABC
ACB
BAC
BCA
CBA
CAB

= 6 routes
= 3!

salesperson has to travel from home to cities A, B, and C
what are all possible routes?
which one is most efficient?

$O(n!)$