Section 4.3: Logarithmic Growth

Tuesday, October 24, 2023 3:27 PM

suppose you need to find the position of a particular entry in an ordered list:

[12, 13, 27, 35, 52, 71, 89]

where is 52?

method #1: start at left and look at each entry in the list until you get to the entry of interest

→ this method is O(n) and is called a linear search

method #2: look at the entry in the middle of the list. If it's the entry of interest (52), stop! If it's not, is the entry of interest greater than or less than the middle entry?

If greater than, then discard the bottom half of the list and repeat.

[12, 13, 27, 35, 52, 71, 89]

is 52 = 35? no!
is 52 > 35? yes!

[52 71 89]

is 52 = 71? no!
is 52 > 71? no!

[52]

↑ is 52 = 52 ? yes! STOP!

for method #2, if your list has one million entries, you need a maximum of 20 searches to find your entry of interest

essentially, we are solving

$$2^n = 1\,000\,000$$

this requires a new function called a logarithm

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logarithms:

if $2^3 = 8$, then $3 = \log_2 8$

the logarithm asks what exponent on the base gives the other number

$\log_{\text{base}}$ other number

examples: $\log_2 16 = 4$

$\log_2 2 = 1$

$\log_3 9 = 2$

$\log_9 3 = \frac{1}{2}$ or 0.5

for calculators if the base is not

$\log_{10} 1000 = 3$

$\log_{10} 0.1 = -1$

if the base is not specified, it's base 10

$\log_{10} 0.1 = -1$

unfortunately, this is not the universal default

in computing, the default is base e

↑ fifth letter of the alphabet

$e = 2.718281828 4905\ldots$

(irrational, like π and $\sqrt{2}$)

in many programming languages

log means $\log_e$

log10 means $\log_{10}$

Java:

Math.log(value)

Math.log10(value)

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to calculate logarithms with a calculator

(most calculators only do base 10 and base e)

$\log_e \rightarrow$ ln for natural log

if we are solving $2^n = 1\,000\,000$,

then $n = \log_2(1\,000\,000)$

$$= \frac{\log 1\,000\,000}{\log 2} = \frac{\ln 1\,000\,000}{\ln 2}$$

$$= \frac{\log 1000000}{\log 2} = \frac{\ln 1000000}{\ln 2}$$

$\approx 19.93 \rightarrow$ round up to 20

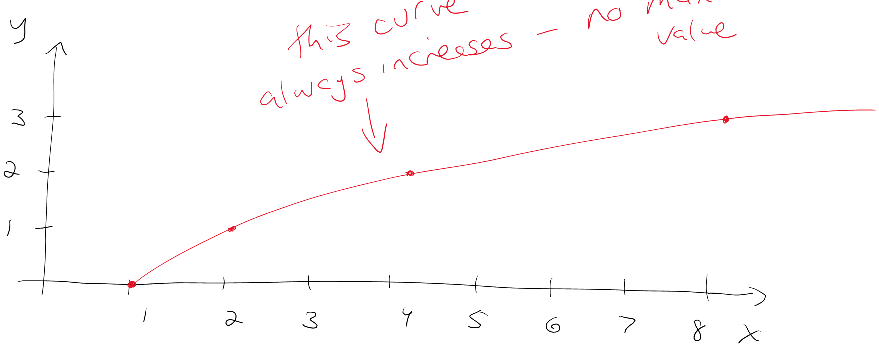
So method #2, which is called a binary search, is $O(\log n)$

Section 4.3: cont'd 2023/10/25

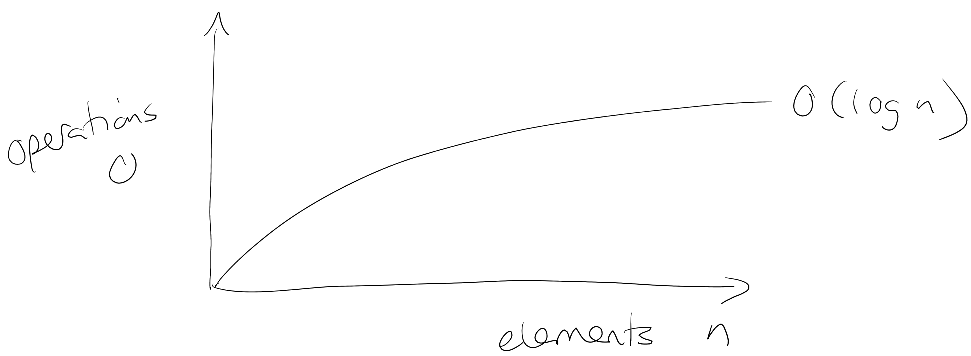
What does the graph of a logarithm look like?

x	$y = \log_2 x$
1	0
2	1
4	2
8	3

this curve always increases – no maximum value



What you need to know for this class



- is always increasing without limit

- doesn't have a maximum value
  - will always eventually be larger than any particular

- will always eventually be larger than any particular value

what about O(n log n) ("linearithmic")