Chapter 8: Probability

Wednesday, November 15, 2023 10:49 AM

Section 8.1: Counting Techniques

example: How many 4-digit positive integers are evenly divisible by 5?

answer: 1000, 1005, 1010, ..., 9995

note: this is an arithmetic sequence with $d = 5$

\# choices for digits:

$$\underline{9}\ \ \underbrace{\underline{10}\ \ \underline{10}}_{\text{choose from 0 to 9}}\ \ \underline{2}$$

9: choose from 1 to 9

2: choose from 0 or 5

now multiply these choices together to get

$$\#\text{choices} = 9 \cdot 10 \cdot 10 \cdot 2 = 1800$$

note: this method only works when you rule out possibilities in one or more slots

works for "divisible by 5" or 2 or 10

doesn't work for "divisible by 3" or 7 etc

multiplication rule:

suppose we have an event which is made up of n different independent steps total number of ways the event can happen = ____ × ____ × ____ × ... × ____

↑ number of ways the first step can happen

↑ 2nd step

↑ final step

example: How many different BC licence plates for cars are there?

(assume that all letters and numbers are used and ignore reserved words and personalized plates)

patterns:

LLL	NNN	L = letter
NNN	LLL	N = number
LLN	NNL	

answer: top pattern:

$$\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}$$

$$= 26^3 \cdot 10^3$$
$$= 17\ 576\ 000$$

total plates for all patterns $= 3(17\ 576\ 000)$
$= 52\ 728\ 000$

example: In the mythical Canadian province of Gondor, licence plates follow the pattern letter-letter-letter number-number. Due to recent political events, the letter combination EYE is no longer allowed. How many legal licence plates are there in Gondor?

answer: number of legal plates = total number − number of illegal pl...

answer: number of legal plates = total number − number of illegal plates

total number possible = $\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 26^3 \cdot 10^2$

illegal plates : $\underset{\text{"E"}}{\underline{1}}\ \underset{\text{"Y"}}{\underline{1}}\ \underset{\text{"E"}}{\underline{1}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 100$

number of legal plates = $26^3 10^2 - 1^3 \cdot 10^2 = 1\ 757\ 500$

note: the reason that you can't just say
$\underline{25}\ \underline{25}\ \underline{25}\ \underline{10}\ \underline{10}$

is that you can still start with an E
(ECR 58 is okay)

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tip: when finding the number of allowed outcomes, sometimes it's easier to calculate the total number of outcomes and subtract from those the outcomes that are not allowed

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the addition rule:

example: how many positive integers from 1 to 20 inclusive are

a) evenly divisible by 2?
b) " " " 3?
c) " " " 2 or 3?

answer: brute force method

divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 10 divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 — 10

divisible by 3: 3, 6, 9, 12, 15, 18 — 6

divisible by 2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 — 13

note: $13 \neq \underbrace{10 + 6}_{\text{contains duplicates}}$

but $13 = \underset{\substack{\uparrow \\ \text{first group}}}{10} + \underset{\substack{\uparrow \\ \text{second group}}}{6} - \underset{\substack{\uparrow \\ \text{overlap}}}{3}$

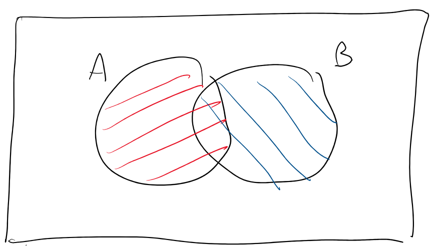
but

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

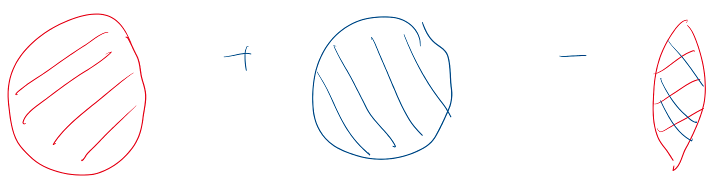
$n(A \text{ or } B)$: number of events containing A or B

$n(A \text{ and } B)$: can also write $n(AB)$

Venn diagrams



shaded area =



double-counted

example: How many 4-digit PINs (personal identification number)

a) start with 9

personal identification number

a) start with 9
b) end in 4
c) start with 9 or end in 4?
d) start with 9 or 4?

answer: a) start 9:: $\underset{9}{\underline{1}}\ \underbrace{\underline{10}\ \underline{10}\ \underline{10}}_{\text{any digit}} = 10^3$ or 1000

b) end 4: same

c) start with 9 or end 4

$n(\text{start 9 or end 4}) = n(\text{start 9}) + n(\text{end 4}) - n(\text{both})$

$$\boxed{n(\text{both}) = \underset{9}{\underline{1}}\ \underset{\text{any}}{\underline{10}}\ \underset{\text{any}}{\underline{10}}\ \underset{4}{\underline{1}} = 10^2 \text{ or } 100}$$

$n(\text{start 9 or end 4}) = 1000 + 1000 - 100$
$= 1900$

d) $n(\text{start with 9 or 4}) = n(\text{start 9}) + n(\text{start 4}) - n(\text{both})$
$= 1000 + 1000 - 0$
$= 2000$

or: $\underset{9 \text{ or } 4}{\underline{2}}\ \underbrace{\underline{10}\ \underline{10}\ \underline{10}}_{\text{any}} = 2000$

example: How many 4-digit PINs are there if repetition of digits is not allowed?

answer: $\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = 5040$

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digression: will not be tested

there's! another! way! to! calculate! this!

$$10 \cdot 9 \cdot 8 \cdot 7 = 10 \cdot 9 \cdot 8 \cdot 7 \cdot \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{6!}$$

number of choices = 10 (digits)

number of slots = 4

denominator $(10-4)!$

this is called a permutation and on your calculator, it's the $_nP_r$ button

so enter $10P4$ to get 5040

Section 8.1: cont'd

example: How many 5-character case-sensitive alphanumeric passwords are there

a) in total?

b) that contain at least one letter and at least one number?

answer: a) how many choices do we have for characters?

alphanumeric - letters and numbers

case sensitive - uppercase and lowercase letters are considered to be different (A vs a)

choices = 10 digits + 26 uppercase + 26 lowercase
= 62

$\underline{62}\ \underline{62}\ \underline{62}\ \underline{62}\ \underline{62} = 62^5$ or 916 132 832

b) want at least one letter <u>and</u> at least one number

total allowed = total possible − total not allowed

not allowed: all numbers: 10^5
all letters: 52^5

total allowed = $62^5 - 10^5 - 52^5$
= 535 828 800
or 5.35×10^8

one common mistake:

$\frac{52}{10}\ \frac{10}{52}\ \underline{62}\ \underline{62}\ \underline{62}$

still leaving out other valid passwords
like AAAA2