Section 8.2: Classical Probability

Tuesday, November 21, 2023 3:49 PM

Classical probability: if all outcomes are equally likely, then the probability of event E happening is:

$$P(E) = \frac{n(E)}{n_{tot}}$$

$n(E)$ ← number of ways E can happen

n_{tot} ← total number of outcomes

"P of E" probability of E happening

example: If you roll two fair 4-sided dice, what's the probability that the sum of the rolls is 3 or less?

fair = all rolls are equally likely (not fair = loaded or weighted)

note: one die, two dice (dice is plural)

sample space: ← set of all possible outcomes

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

16 outcomes

$$P(\text{sum} \le 3) = \frac{n(\text{sum} \le 3)}{n_{tot}} = \frac{3}{16}$$

what, then, is the probability that the sum will be greater than 3?

$$P(\text{sum} > 3) = 1 - P(\text{sum} \le 3)$$
$$= 1 - 3/16$$
$$13/$$

$$= 1 - 3/16$$
$$= 13/16$$

why? $P(A) = 1 - P(\bar{A})$

what is the probability of rolling a sum of 5?

$$P(\text{sum}=5) = \frac{n(\text{sum}=5)}{\text{total}} = \frac{4}{16} = \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$$

what is the probability that at least one die show the number 3?

$$P(\text{at least one } 3) = \frac{7}{16}$$

for event A, the complement can be written as

$$\bar{A}$$

other notations: A^c, A', $\sim A$, $\neg A$

the complement of A is the set of outcomes in which A does not occur

$$P(A) + P(\bar{A}) = 1 \text{ or } 100\%$$

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probability demonstration with two 4-sided dice

what are the rolls of individual dice

what is the sum of the dice?

individual dice

1	~~||||~~ ||||
2	~~||||~~ ||
3	~~||||~~ |||
4	~~||||~~ |

2	|
3	||
4	~~||||~~
5	||
6	||
7	|||
8	

Some vocabulary:

experiment – process by which an observation (measurement) is obtained

example: you roll a six-sided die

simple event – the outcome observed on a single repetition of the experiment

example: you roll a 2

compound event – a collection of simple events (sometimes just called an event)

example: you roll an even number

sample space – the complete list of all simple events (all possible outcomes)

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two properties of probability:

(1) $0 \le P(A) \le 1$

② the sum of the probabilities for all possible outcomes is one

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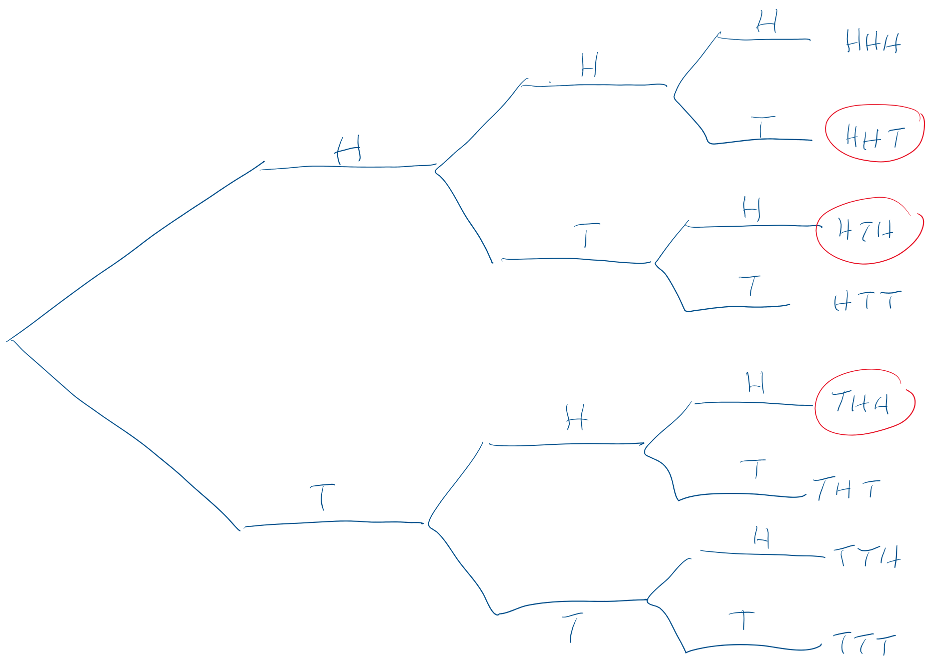
if you are having trouble generating the sample space, another approach is a tree diagram:

example: write out the sample space for flipping a coin three times and recording the result.



note: how many ways can you get only one tail?

if the coin is fair, what's the probability that in three flips, you'll get exactly one tail? 3/8

example: At the Red Barn Market, you can get an ice-cream cone with two scoops of ice cream. Let's assume that you have to choose two different flavours for your scoops and that which flavour is on top doesn't matter. Let's further assume that when averaged over all customers, each flavour is equally likely.

flavours available: chocolate
vanilla
strawberry
blueberry

a) How many different ice cream cones are possible?

brute force: CV VS SB
CS VB
CB

b) what's the probability that a random customer will order chocolate as one of the two scoops?

$$P(C) = \frac{n(C)}{n_{tot}} = \frac{3}{6} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

c) whats the probability that a random customer orders chocolate and vanilla?

$$P(CV) = \frac{n(CV)}{n_{tot}} = \frac{1}{6}$$

d) what's the probability that a random customer orders chocolate or vanilla?

$$P(C \text{ or } V) = \frac{n(C \text{ or } V)}{n_{tot}} = \frac{5}{6}$$

e) calculate (d) again using a different method

$$P(C \text{ or } V) = 1 - P(\overline{C \text{ or } V})$$
$$= 1 - P(SB)$$
$$= 1 - \tfrac{1}{6} = \tfrac{5}{6}$$

f) calculate (d) again using <u>yet</u> another method!

$$P(C \text{ or } V) = P(C) + P(V) - P(CV)$$
$$= \tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{6}$$
$$= 5/6$$

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summary of rules:

$$P(\text{event}) = \frac{n(\text{event})}{n_{tot}}$$

$$P(\text{event}) = 1 - P(\overline{\text{event}})$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

this can be zero - then the events A and B are called "mutually exclusive"

note: for questions in which the sample space is small, the brute force method of writing out the complete sample space is perfectly acceptable

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contingency table:

example: suppose we survey students in Math 156 to find out whether they like coffee and/or spicy food

C	$\bar{C}$
like coffee	don't like coffee

		C like coffee	$\bar{C}$ don't like coffee	
S	like spicy food	13	5	18
$\bar{S}$	don't like spicy food	2	4	6
		15	9	24

a) what's the probability that a randomly chosen student likes coffee and spicy food?

$$P(CS) = \frac{n(CS)}{n_{tot}} = \frac{13}{24}$$

b) what's the probability that a randomly chosen student likes coffee?

$$P(C) = \frac{n(C)}{n_{tot}} = \frac{15}{24} = \frac{5}{8}$$

c) what's the probability that a randomly chosen student doesn't like coffee?

$$P(\bar{C}) = 1 - P(C) = 1 - 5/8 = 3/8$$

d) what's the probability that a randomly chosen student likes coffee or spicy food?

	C	$\bar{C}$
S	13	5
$\bar{S}$	2	4

method #1:

$$P(C \text{ or } S) = \frac{n(C \text{ or } S)}{n_{tot}} = \frac{13+5+2}{24} = \frac{20}{24} = \frac{5}{6}$$

method #2:

$$P(C \text{ or } S) = P(C) + P(S) - P(CS)$$

$$= \frac{15}{24} + \frac{18}{24} - \frac{13}{24} = \frac{20}{24} = \frac{5}{6}$$

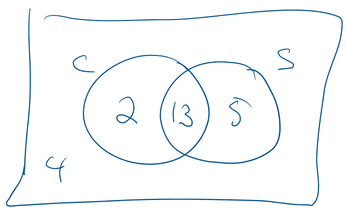
$$= \frac{15}{24} + \frac{18}{24} - \frac{13}{24} = \frac{20}{24} = \frac{5}{6}$$

method #3:

$$P(C \text{ or } S) = 1 - P(\overline{C \text{ or } S})$$
$$= 1 - P(\bar{C}\bar{S})$$
$$= 1 - \frac{4}{24} = \frac{20}{24} = \frac{5}{6}$$

note:

	C	$\bar{C}$
S	13	5
$\bar{S}$	2	4



example: In a class of 45 students, 26 have jobs and 17 have cars. Of those who don't have a car, 10 have jobs.

a) Fill out a contingency table for this situation.

		J jobs	$\bar{J}$ no jobs	
C	car	16	1	17
$\bar{C}$	no car	10	18	28
		26	19	45

45 total number of students

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b) Find the probability that a student has a car or a job.

$$P(C \text{ or } J) = \frac{n(C \text{ or } J)}{n_{tot}} = \frac{16+10+1}{45}$$

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$$= \frac{27}{45} = \frac{3}{5} \text{ or } 0.6 \text{ or } 60\%$$

c) Find the probability that a student has a car but not a job

$$P(C\bar{J}) = \frac{n(C\bar{J})}{n_{tot}} = \frac{1}{45} \quad \text{or} \quad 2.\bar{2}\%$$

note: we could also represent this contingency table using percentages:

	J	$\bar{J}$	
C	35.6%	2.2%	37.8%
$\bar{C}$	22.2%	40.0%	62.2%
			100%