Term: 2022 Name: Solution Set

Instructor: Patricia Wrean

MATH 156-X01
Practice Test 3B

Total = $\frac{\quad}{30}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app.

GOOD LUCK!

1. (4 points) Consider the following.

$$\begin{cases} a_1 = 12 \\ a_n = 7 + a_{n-1} & \text{for } n \geq 2 \end{cases}$$

(a) Is this formula recursive or general? Choose one: recursive / general (1)

(b) Calculate the first three terms: 12 (1), 19 (1), 26 (1)

(-1) for 19, 26, 33

2. (6 points) Consider the following:

$$288, 144, 72, 36, 18, 9$$

(a) Give a general formula for a_n. Be sure to specify what values to use for the index. Draw a box around your answer.

(-½) no box

$a_n = a_m r^{n-m}$ (1)

(½ each)

$a_n = 288\left(\frac{1}{2}\right)^n$ (1) for $0 \leq n \leq 5$

or $a_n = 288\left(\frac{1}{2}\right)^{n-1}$ for $1 \leq n \leq 6$

(b) Give a recursive formula for a_n. Be sure to specify what values to use for the index. Draw a box around your answer.

$\begin{cases} a_0 = 288 \text{ (1)} \\ a_n = \frac{1}{2} a_{n-1} \text{ (1) for } 1 \leq n \leq 5 \end{cases}$ (½ each)

or

$\begin{cases} a_1 = 288 \\ a_n = \frac{1}{2} a_{n-1} \text{ for } 2 \leq n \leq 6 \end{cases}$

3. (4 points) Consider the following:

$$13 + 16 + 19 + \ldots$$

(a) Circle one: this is arithmetic / geometric / neither

(b) Circle one: this is finite / infinite

(c) Calculate S_3. 48

(d) Calculate S_5. 95

$S_3 = 13 + 16 + 19$

$S_5 = 13 + 16 + 19 + 22 + 25$

4. (5 points) Consider the following.

$$\sum_{n=4}^{28} 3^{n-2}$$

(a) Is this a sequence or a series? Choose one: sequence / series (1)

(b) How many terms does it have? $k = n - m + 1 = 28 - 4 + 1$ 25 (1)

(c) Calculate the sum. Show your work below. 3.8×10^{12} (3)

$$\sum_{n=4}^{28} 3^{n-2} = \underset{(4)}{3^2} + \underset{(5)}{3^3} + \underset{(6)}{3^4} + \ldots$$

$$= 9 + 27 + 81 + \ldots \quad (1)$$

geometric with $r = 3$

$$S_k = \frac{a_m(1 - r^k)}{1 - r} = \frac{9(1 - 3^{25})}{1 - 3} \approx 3.8 \times 10^{12}$$

(1) (1)

5. (3 points) Label the following as "arithmetic", "geometric" or "neither".

   (a) $1, 8, 27, 64, \ldots$ neither

   (b) $11, 7, 3, -1, \ldots$ $d = -4$ arithmetic

   (c) $12, -24, 48, \ldots$ $r = -2$ geometric

6. (2 points) If you look up algorithms on how to sort a list, you will find that in terms of operations, Bubblesort has $O(n^2)$ while Heapsort has $O(n \log n)$.

   Based only on this information, which method is more efficient for large values of n? Indicate the correct choice.

   (a) Heapsort
   (b) Bubblesort
   (c) They both have the same efficiency

   Why?

   (a) Because n^2 grows faster than $n \log n$ and bigger is better.
   (b) Because n^2 and $n \log n$ grow at the same rate.
   (c) Because $n \log n$ grows slower than n^2 and fewer operations means that the program will run faster.
   (d) There is not enough information to decide.

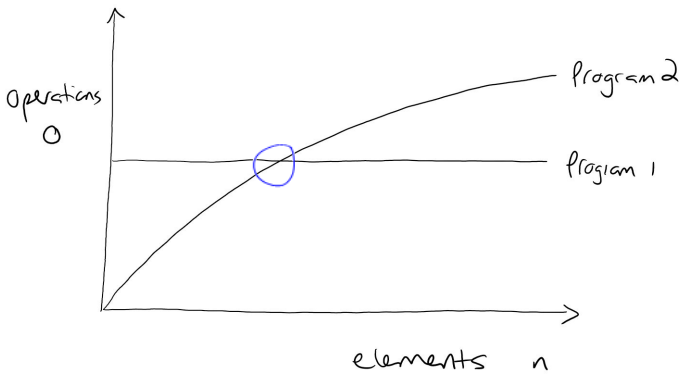
7. (3 points) Evaluate the following logarithms.

   (a) $\log_8(64)$ $8^2 = 64$ 2

   (b) $\log_{10}(0.1)$ $10^{-1} = 0.1$ -1

   (c) $\log_3(1)$ $3^0 = 1$ 0

8. (3 points) The following graph shows the number of operations O required to complete a task of size n for Programs 1 and 2. The number of operations required for Program 1 is a constant, so Program 1 is a horizontal straight line.



   Indicate whether the following statements are true or false by selecting the correct answer.

   (a) It's possible that for a certain value of n, the two programs are equally efficient. (True) / False

   (b) Program 2 is a better choice than Program 1 for some circumstances. (True) / False

   (c) If Program 2 is $O(\log n)$, then for large values of n it could curve downwards and become more efficient than Program 1. True / (False)