Term: Winter 2024

Instructor: Patricia Wrean

Name: Solution Set

MATH 156
Test 3, Version B

Total = $\overline{}$ 25

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app. You may not share calculators between students.

GOOD LUCK!

1. (3 points) Label the following as "arithmetic", "geometric" or "neither".

   (a) $2, \overset{\times 7}{14}, \overset{\times 7}{98}, \ldots$ geometric

   (b) $36, \overset{-9}{27}, \underset{-9}{18}, \ldots, -108$ arithmetic

   (c) $1, 8, 27, 64, \ldots$ neither

2. (4 points) Consider the following list of numbers:

$$102, 72, 42, \ldots$$ arithmetic with $d = -30$ ½

   (a) Give a general formula for a_n. Be sure to specify what values to use for the index, and simplify your answer. Draw a box around your answer.

$$a_n = a_m + (n-m)d$$

if you choose $m=1$:

$$a_n = 102 + (n-1)(-30)$$
$$= 102 - 30n + 30$$
$$\boxed{\underbrace{a_n = 132 - 30n}_{1} \quad \underbrace{\text{for } n \geq 1}_{½}}$$

if you choose $m=0$

$$a_n = 102 + n(-30)$$
$$\boxed{a_n = 102 - 30n \text{ for } n \geq 0}$$

   (b) Give a recursive formula for a_n. Be sure to specify what values to use for the index. Draw a box around your answer.

$$\begin{cases} a_1 = 102 & ½ \\ \underset{1}{a_n = a_{n-1} - 30} \quad \underset{½}{\text{for } n \geq 2} \end{cases}$$

3. (2 points) Consider the following.

$$\begin{cases} a_0 = 12 \\ a_n = 5a_{n-1} \qquad \text{for } n \geq 1 \end{cases}$$

Calculate the first three terms: 12 , 60 , 300

$a_0 = 12$
$a_1 = 5a_0 = 5(12) = 60$
$a_2 = 5a_1 = 5(60) = 300$

$\boxed{-1 \text{ for } 12, 300, 1500}$

4. (4 points) Consider the following.

$$4(5) + 5(6) + 6(7) + \ldots + 69(70)$$

(a) Write this sum using sigma notation.

$$\sum_{n=4}^{69} n(n+1) \quad \text{or} \quad \sum_{n=5}^{70} n(n-1) \quad \text{or} \quad \sum_{n=0}^{65} (n+4)(n+5)$$

(b) How many terms does it have? $k = n - m + 1 = 69 - 4 + 1 =$ 66

(c) Calculate S_5. 220

$S_5 = 20 + 30 + 42 + 56 + 72$

5. (3 points) Consider a list of numbers that starts at a value of 4. Every number after that is equal to the previous number times 3. Find the sum of the first sixty numbers in this list.

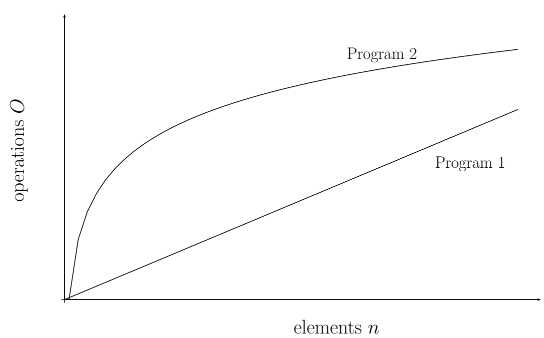
list is 4, 12, 36, ...

want sum of first 60 numbers of $4+12+36+\ldots$

geometric with $r=3$

$$S_k = \frac{a_m(1-r^k)}{1-r} = \frac{4(1-3^{60})}{1-3} \approx \boxed{8.478\times10^{28}}$$

6. (2 points) This graph shows the number of operations O required to complete a task of size n elements for Programs 1 and 2, where Program 1 is a straight line and Program 2 is a curved line.



Indicate whether the following statements are true or false.

(a) Program 1 is $O(n)$. True / False

(b) Program 2 is always a better choice than Program 1. True / False

(c) Program 1 is always a better choice than Program 2. True / False

(d) It's possible that for some value of n, that the two programs are equally efficient. True / False

7. (2 points) Evaluate the following logarithms.

   (a) $\log_{10}(0.01)$ $10^{-2} = 0.01$ $\underline{-2}$

   (b) $\log_7(7)$ $7^1 = 7$ $\underline{1}$

8. (2 points) For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

   (a) $2\log n + 3n$ $\underline{O(n)}$

   (b) $3n! + 5(2^n)$ $\underline{O(n!)}$

9. (3 points) Indicate whether the following statements about the $O(\log n)$ curve are true or false.

   (a) If n gets large enough, the curve of $O(\log n)$ will eventually curve downward. True / False

   (b) No matter how big n is, the curve of $O(\log n)$ will always increase. True / False

   (c) If n gets large enough, the curve of $O(\log n)$ will reach a certain value and stay there. True / False